

Development of VIDAR – a growth model for Danish forest tree species

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Chapter 1

About Vidar

In Norse mythology, Víðarr, which is possibly derived from víðr "forest" and arr "warrior" (often Anglicised VIDAR), is the son of Odin and the giantess Grídr. During Ragnarök, Odin is eaten by the wolf Fenrir, and Víðarr will avenge his death by killing the beast. According to the *Vafrúðnismál*, Víðarr kills Fenrir by stepping on his lower jaw with his foot, on which he wears a shoe made of the pieces from the toe and heel that were cut and thrown away when people's shoes were made. The shoes protected him from being devoured by Fenrir. Having his foot placed, Víðarr will take the wolf's upper jaw and tear the beast apart. However, according to the *Völuspá* Víðarr uses his sword to kill the wolf by driving it straight into the heart. In the reborn world that arises after Ragnarök, Víðarr is preordained to return with his brother Váli. Víðarr is referred to as "the silent son of Odin" in the *Skáldskaparmál* (second part of the *Prose Edda*) (Encyclopædia Britannica, 2007).

During recent years, dynamic stand level models for Norway spruce, oak, beech, and Douglas fir in Denmark have been developed (Leary et al., 1999; Skovsgaard et al., 1999; Johannsen et al., 1999; Johannsen, 1999; Nord-Larsen, 2006b; Christensen et al., 2003). The models are in the present form merely mathematical expressions and estimated parameter values and are not easily accessible for the potential users of such models in practical forestry. Hence, the aim of the VIDAR-project is to

develop a practical and user-friendly tool for the development of growth models that will be distributed along with the periodical SKOVEN and will be freely available on the *Forest & Landscape* homepage.

The basis of VIDAR is the dynamic stand-level growth models developed for beech, oak, Norway spruce, Douglas fir, Sitka spruce, and silver fir. However, VIDAR is dependent on a series of auxiliary functions that describe relationships between different stand variables to aid the use and interpretation of the growth models. Further, as more data and knowledge on dynamic growth models has become available since the development of the first models some of the models have been re-estimated using a common model for all species. This report describes the development and estimation of the dynamic growth models and reports the development and estimation of the additional functions in VIDAR.

Chapter 2

Data

The dynamic stand-level models and auxiliary functions were estimated on permanent sample plot data from the even-aged and mono-specific spacing, species and thinning experiments of *Forest & Landscape*. The data included 69 experiments in beech (201 individual plots), 53 experiments in oak (141 individual plots), 75 experiments in Norway spruce (366 individual plots), 35 experiments in Sitka spruce (77 individual plots), 26 experiments in silver fir (28 individual plots), and 23 experiments in Douglas fir (35 individual plots). Plot sizes varied between 0.0022 and 2.65 ha, with an average ranging from 0.16 to 0.34 ha, depending on species. The experiments were located in most parts of Denmark (figure 2.1) and covered a wide range of different site types and growth conditions although the majority of plots with deciduous species were located in the eastern parts of the country.

2.1 Measurements

The data were collected from 1872 to 2008 and the stands were observed for 10 to 131 years. Despite the long time span, data collection on the permanent sample plots has been remarkably uniform. In the earliest measurements diameter readings were taken at 1/20 tree height. After 1880 trees were generally marked permanently at 1.3 m (until 1902 at 4.15 feet \sim 1.3m), numbered and recorded individually. The conversion to the metric system in 1902 had little influence on the measurement practises except that trees were now recorded in tally lists to 1-cm rather than 1-inch diameter classes. Consequently, the data used for modeling growth and yield represents more than 120 years of consistent measurements. The total number of measurement occasions was 2104 for beech, 1580

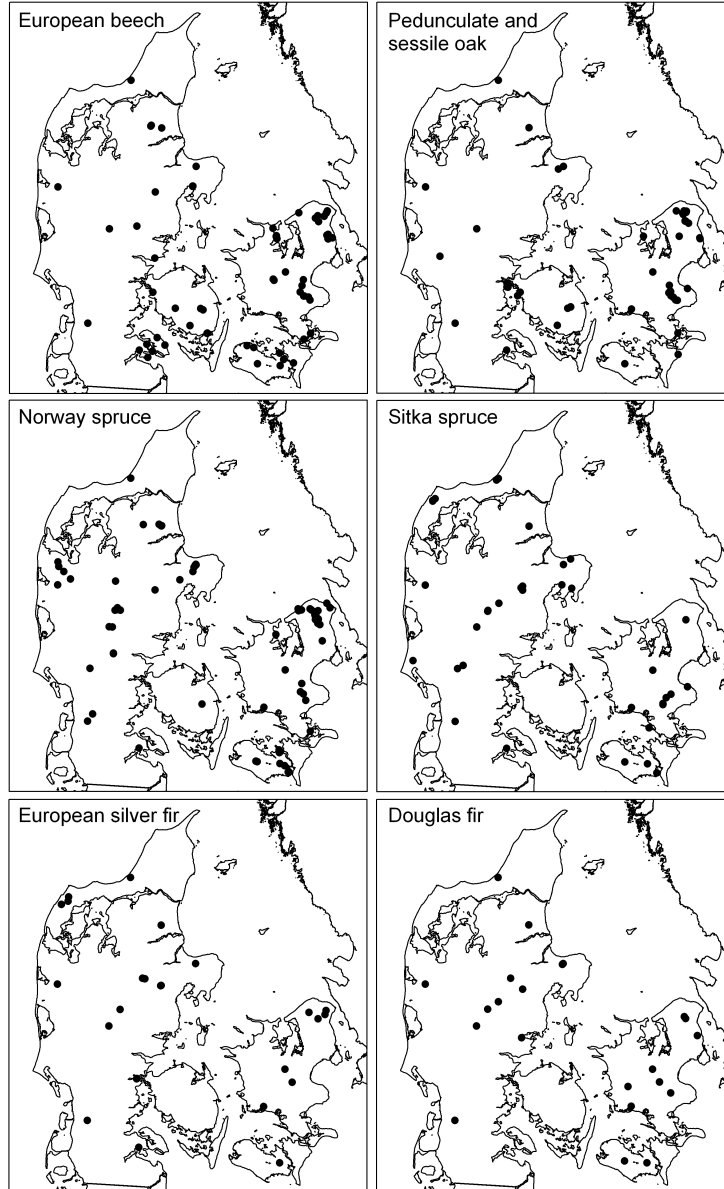


Figure 2.1: Map of the 281 experiments used for estimating site-specific height growth models. European beech: 69 experiments with 201 plots, oak: 53 experiments with 141 plots, Norway spruce: 75 experiments with 366 plots, Sitka spruce: 35 experiments with 77 plots, European silver fir: 26 experiments with 28 plots, and Douglas fir: 23 experiments with 35 plots.

for oak, 3710 for Norway spruce, 793 for Sitka spruce, 213 for European silver fir, and 431 for Douglas fir (please note that the number of measurements in individual model estimation procedures may be less than the total number because of differences in data requirements for the individual procedures).

In the majority of plots, all trees were numbered, marked permanently at breast height (1.3 m) and recorded individually. In some very young stands with high stem numbers, trees were recorded in tally lists to 1-cm diameter classes (before 1902 to 1-inch classes). Also in very young stands with high stem numbers, only a subset of stems were measured, e.g. every fifth or tenth row. Measurements of diameter were obtained by averaging two perpendicular calliper readings taken at breast height (1.3 m) for each tree. Observations also included records on whether the tree was alive or dead at the time of measurement. The height of live trees was measured using a hypsometer and felled trees were measured on the ground using a tape measure. About 30 tree height measurements per plot were obtained on each measurement occasion.

2.2 Calculations

Stem numbers, N (ha^{-1}), were calculated as the number of individual trees per hectare taller than 1.3 m. When trees forked below 1.3 m, each stem was measured individually but multiple stems from the same root were counted as one tree. Within the research plots, trees were typically separated into over- and understorey and the understorey was measured less intensively than the overstorey. Understorey trees were not included in this study.

Stand basal area, G ($\text{m}^2 \text{ ha}^{-1}$), of each plot was estimated by summation of individual tree basal areas calculated from the diameter measurements. When trees were recorded in tally lists, the mid-diameter of each class was used as an estimate of the diameter of all trees in that class. Quadratic mean diameter, D_g (cm), was derived from the estimates of N and G .

Based on paired observations of diameter and height, height-diameter regressions were estimated for each plot and measurement combination using a modified Näslund-equation (Näslund, 1936; Johannsen, 2002):

$$h = 1.3 + \left(\frac{d}{\alpha + \beta \cdot d} \right)^3 \quad (2.1)$$

where d is the diameter at breast height, h is the total tree height and α and β are parameters to be estimated. Subsequently, the equations were used to estimate the height of trees not measured.

Dominant height, defined as the mean height of the 100 thickest trees per hectare (H_{100}), was calculated for each plot and measurement occasion. In the few cases where stem numbers were

less than 100 per hectare, H_{100} was estimated as the mean height. Height corresponding to the mean basal area tree (H_g) was calculated from the specific diameter-height equation by inserting D_g . Table 2.1 presents a summary of the data.

2.3 Data ranges

The data included a wide range of different treatments in terms of initial spacing and subsequent thinning practices, from unthinned control plots to heavily thinned plots. In the thinning experiments, the treatments included A-, B-, C-, and D-grade thinnings, and in some cases even heavier thinnings. Usually, the D-grade is thinned to a basal area of 50% of the unthinned control (A-grade). The B- and C-grades are intermediate, dividing the interval between A- and D-grades equally. Some plots were managed according to other thinning strategies, such as group or selection thinning, and others were managed according to contemporary thinning strategies. The plots were all essentially even-aged and mono-specific. The plots were generally thinned from below with the objective to obtain a homogenous stand, which make the data less suited for modeling the growth of heterogeneous stands or stands thinned according to other principles. However, particularly for broadleaved species, thinning from above does occur, particularly in young stands.

2.3.1 Beech

Stand ages ranged from 14 to 211 years, but the majority of measurement occasions were in stands from 20 to 100 years old (Figure 2.2). The number of measurements in very young (<20 years) and very old (>150 years) stands were limited.

For beech, measured stands were located mainly on relatively good sites in the eastern parts of Denmark. Site classes ranged from 11.8 to 28.3 m (dominant height at age 50 years), but the majority of stands were in site classes ranging from 20 to 24 m, which may be characterized as good beech sites. Consequently, the models derived from this data may not perform particularly well on relatively poor sites.

Treatments ranged from heavily thinned stands with basal areas of less than 10 m²ha⁻¹ to unthinned control plots with more than 50 m²ha⁻¹. However, the vast majority of stands were thinned moderately, having basal areas of 20-30 m²ha⁻¹.

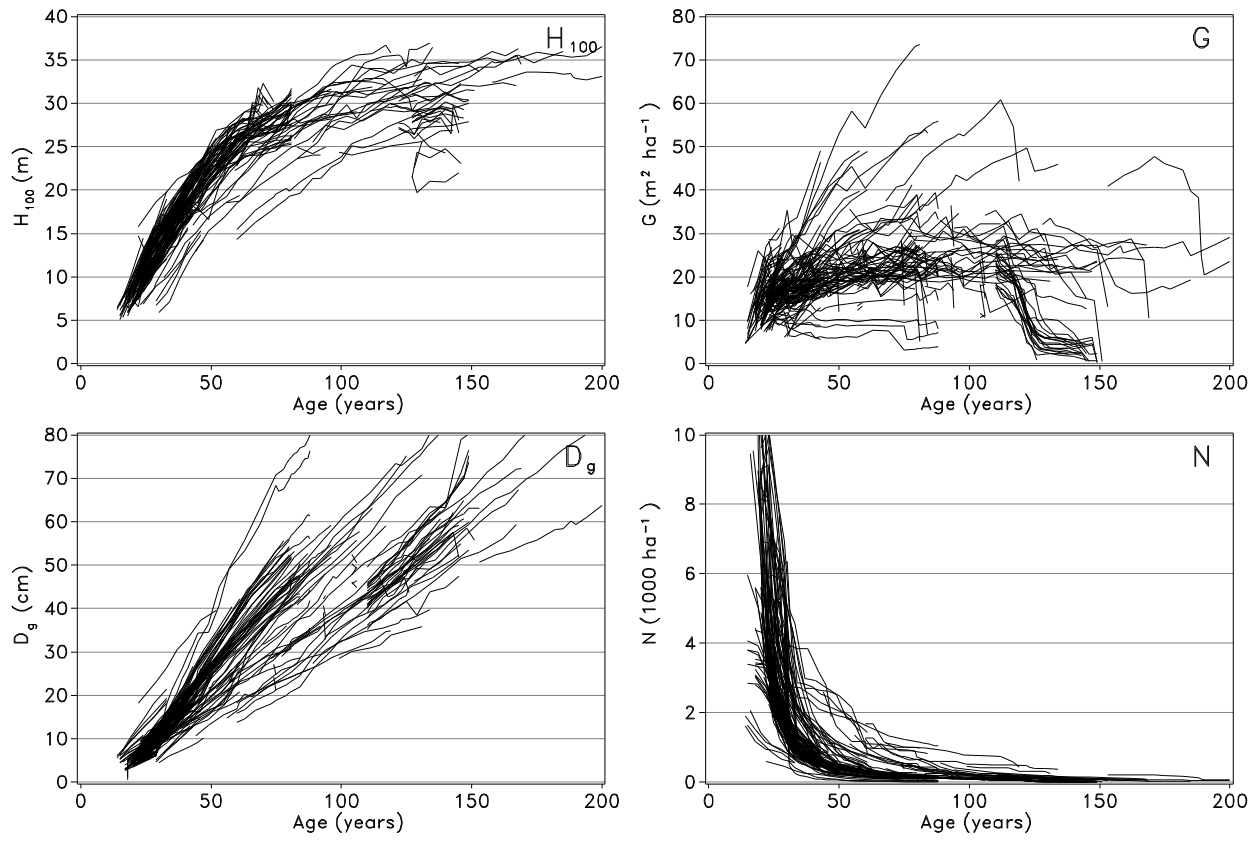


Figure 2.2: Stand-level values of H_{100} , G , D_g , and N for beech.

2.3.2 Oak

Stand ages ranged from 11 to 184 years, but the majority of measurement occasions were in stands aging from 25 to 100 years (Figure 2.3). The number of measurements in very young (<20 years) and very old (>150 years) stands were relatively limited.

For oak, site classes ranged from 7.1 to 21.4 m (height at age 50 years), but the majority of stands were in site classes ranging from 17 to 21 m, which may be characterized as good oak sites. Consequently, the models derived from this data may not perform equally well on poor sites.

In accordance with common thinning practices for oak in Denmark, the majority of stands were quite heavily thinned with basal areas ranging between 10 and 20 m²ha⁻¹. Less intensive thinning practices were applied in some experiments including some unthinned control plots with basal areas of 20-40 m²ha⁻¹.

2.3.3 Norway spruce

Stand ages of the Norway spruce data ranged from 14 to 104 years, but the majority of measurements were carried out in 20 to 60 years old stands (Figure 2.4).

The data on Norway spruce represents a wide range of site classes from 8.7 – 27.6 m (dominant height at age 50). Considering stand height growth the Norway spruce data falls into two groups: 1) stands of first generation Norway spruce on sandy, former heathlands or 2) stands on sandy or clayey moraine or subsequent generations of Norway spruce on sandy, former heathlands. The former group is characterized by a slow start due to frost and wind and a subsequent poor height growth.

Treatments ranged from heavily thinned stands with basal areas less than 15 m²ha⁻¹ to unthinned control plots with more than 50 m²ha⁻¹. However, the vast majority of stands were treated moderately, with basal areas of 20-40 m²ha⁻¹.

2.3.4 Sitka spruce

In sitka spruce stand ages ranged from 16 to 105 years, but the majority of measurements were carried out in stands aging 25 to 50 years and only few measurements were carried out in stands older than 75 years (Figure 2.5). Site classes ranged from 17.1 to 31.1 m (dominant height at age 50 years).

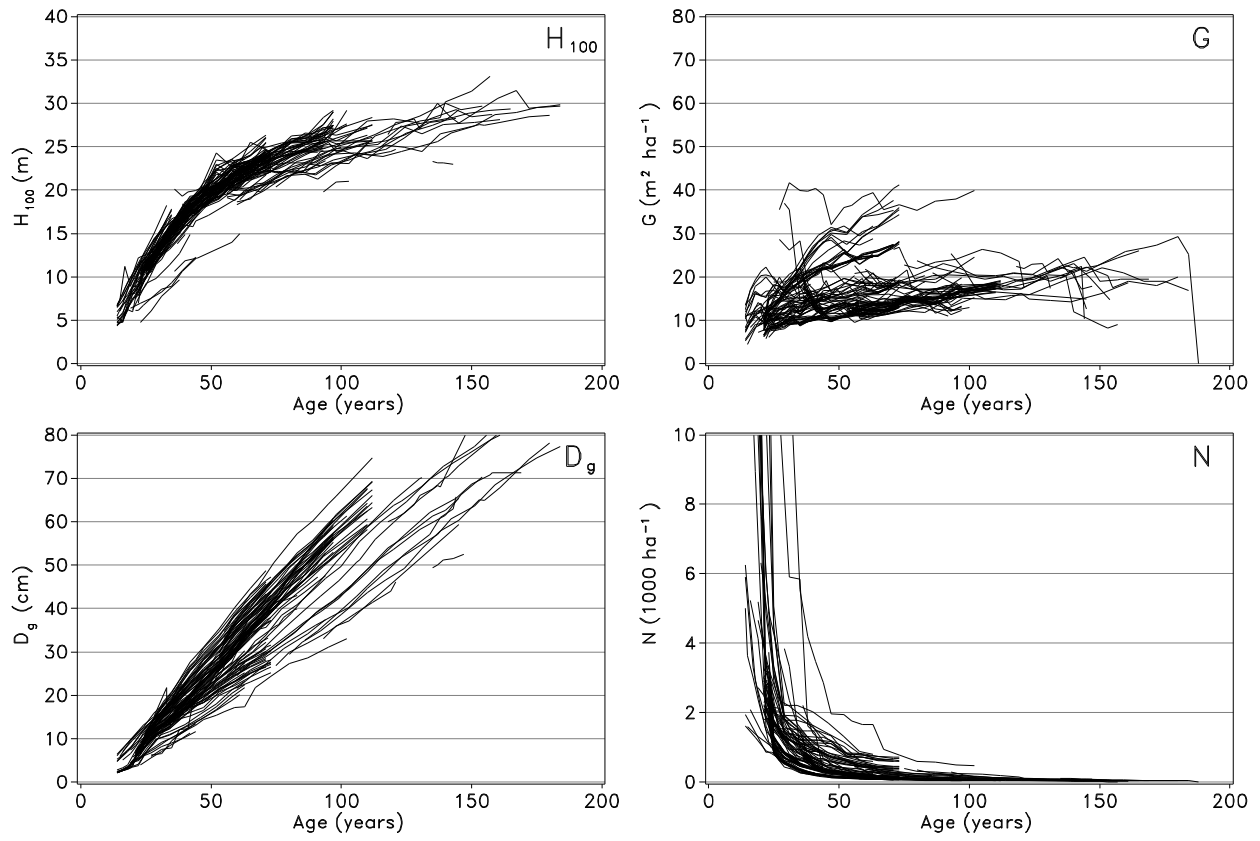


Figure 2.3: Stand-level values of H_{100} , G , D_g , and N for oak.

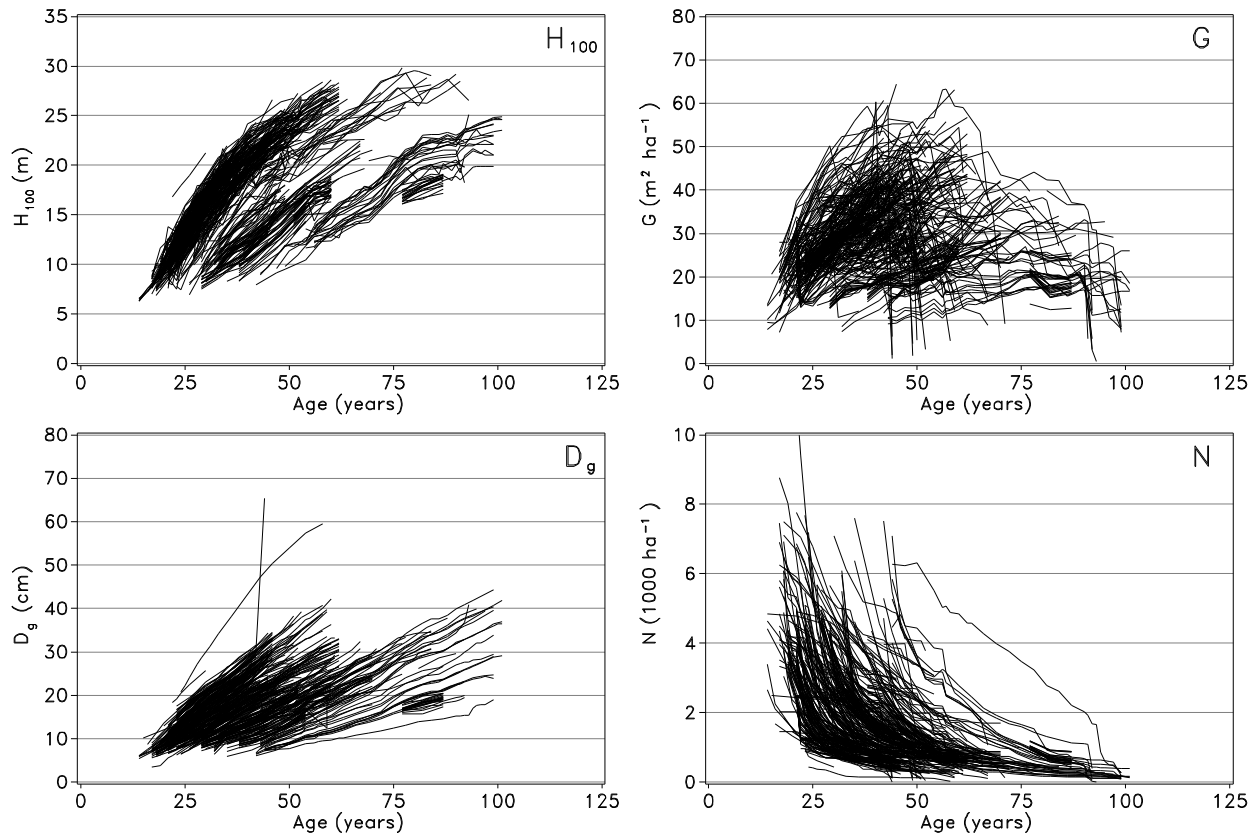


Figure 2.4: Stand-level values of H_{100} , G , D_g , and N for Norway spruce.

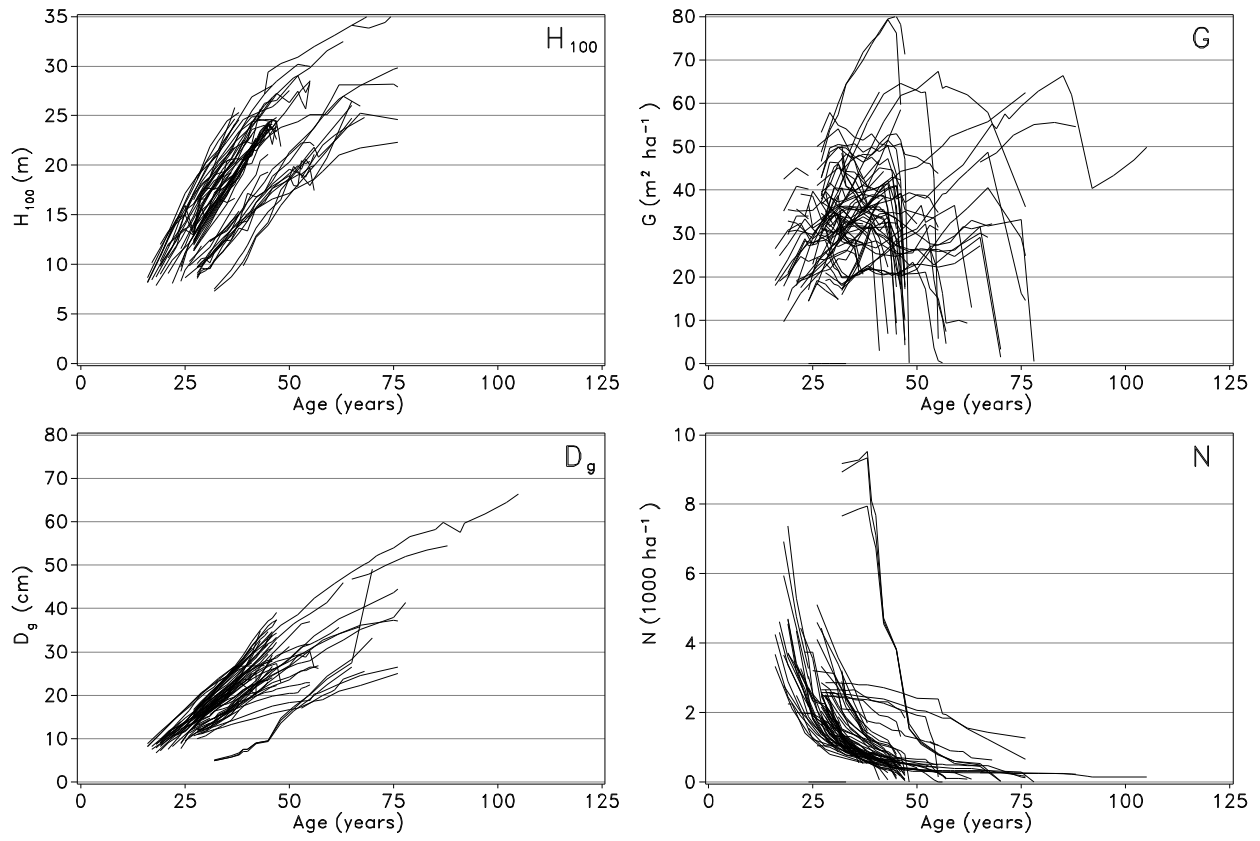


Figure 2.5: Stand-level values of H_{100} , G , D_g , and N for Sitka spruce.

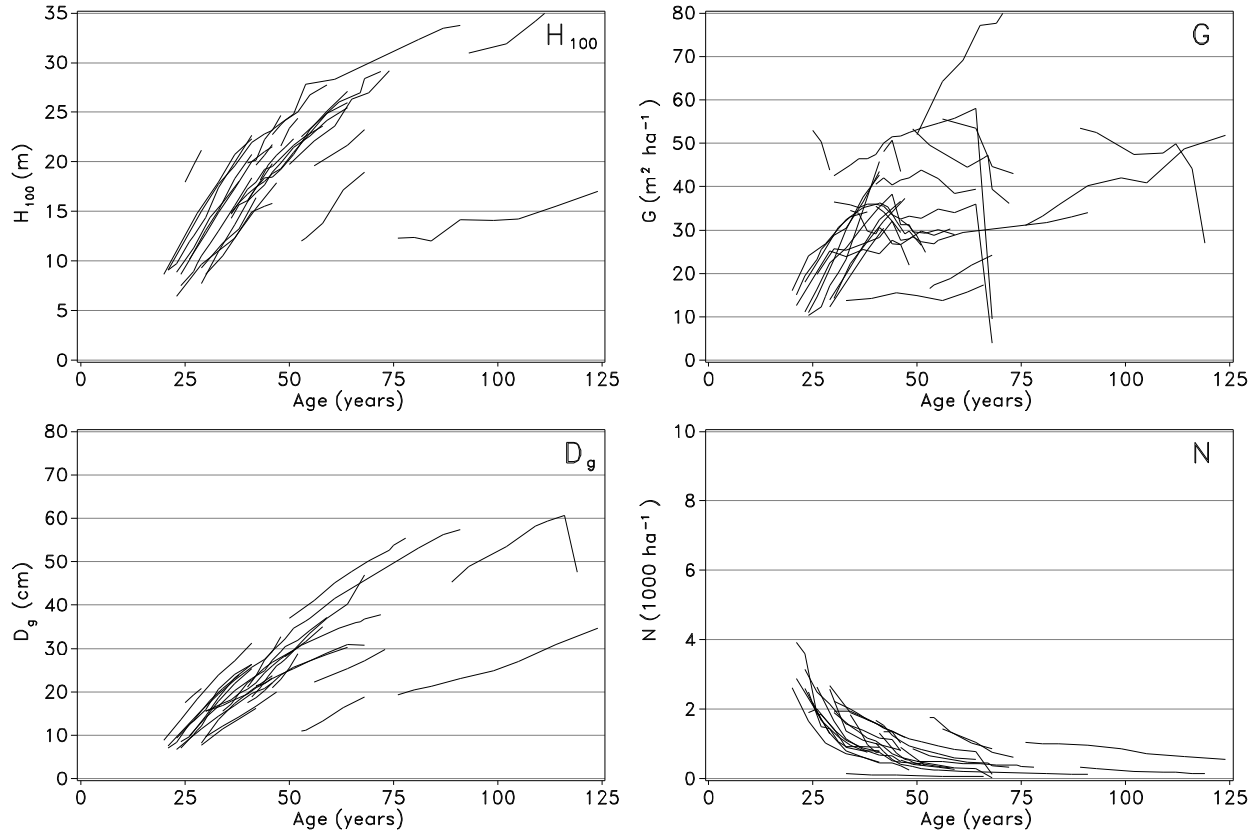


Figure 2.6: Stand-level values of H_{100} , G , D_g , and N for silver fir.

2.3.5 Silver fir

In silver fir stand ages ranged from 20 to 124 years, but the majority of measurements were carried out in stands aging 25 to 50 years and only few measurements were carried out in stands older than 75 years (Figure 2.6). Site classes ranged from 17.1 to 31.1 m (dominant height at age 50 years).

2.3.6 Douglas fir

Stand ages ranged from 11 to 120 years, but the majority of measurement occasions were in stands from 15 to 50 years old (Figure 2.7). The number of measurements in old (>75 years) stands was relatively limited.

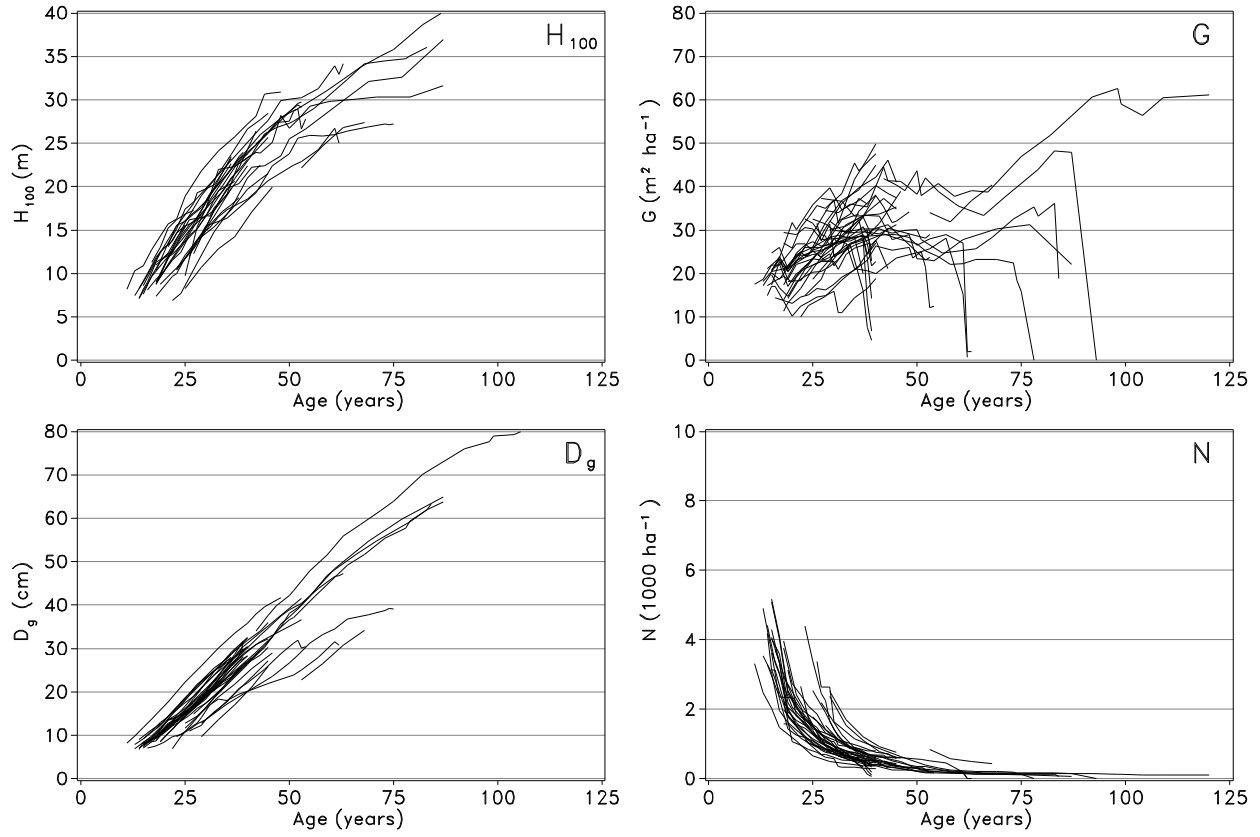


Figure 2.7: Stand-level values of H_{100} , G , D_g , and N for Douglas fir.

Site classes ranged from 18.3 to 32.7 m (dominant height at age 50 years). A relatively wide range of basal areas is represented in the data (20-60 $\text{m}^2 \text{ha}^{-1}$), but the majority of plots were thinned to basal areas ranging from 20 to 40 $\text{m}^2 \text{ha}^{-1}$.

Table 2.1: Summary statistics of dominant height (H_{100}), age (T), quadratic mean diameter (D_g), stem number (N) and basal area (G). Please note that the number of measurements and the data ranges in individual model estimation procedures may be less than the total number because of differences in data requirements for the individual procedures.

Species	Variable	Unit	n	Mean	Minimum	Maximum	Std. Dev.
Beech	H_{100}	m	1728	21.29	5.08	36.95	7.75
	T	years	2104	67.34	14	211	39.99
	D_g	cm	2104	29.74	0.50	90.57	19.00
	N	ha ⁻¹	2104	1286.61	0.49	24720.45	2181.47
	G	m ² ha ⁻¹	2104	20.35	0.21	73.58	9.30
Oak	H_{100}	m	1443	18.61	4.38	33.08	6.06
	T	years	1580	55.37	11	184	32.07
	D_g	cm	1580	28.29	2.19	91.88	18.09
	N	ha ⁻¹	1580	1604.84	13.89	35032.12	4224.62
	G	m ² ha ⁻¹	1580	15.87	4.46	41.18	5.45
Norway spruce	H_{100}	m	3100	17.53	6.28	29.82	4.79
	T	years	3710	44.67	14	104	18.16
	D_g	cm	3710	18.44	3.46	65.44	7.14
	N	ha ⁻¹	3710	1619.92	4.20	26503.70	1437.81
	G	m ² ha ⁻¹	3710	28.93	0.55	64.41	9.55
Sitka spruce	H_{100}	m	709	18.82	6.66	40.40	6.27
	T	years	793	39.75	16	105	13.37
	D_g	cm	793	21.63	4.93	66.43	9.40
	N	ha ⁻¹	793	1498.27	4.37	9513.52	1406.31
	G	m ² ha ⁻¹	793	34.24	0.18	80.10	13.00
Silver fir	H_{100}	m	179	18.84	6.47	36.54	6.16
	T	years	213	50.16	20	124	21.63
	D_g	cm	213	25.33	6.72	60.63	11.59
	N	ha ⁻¹	213	954.56	3.71	3921.82	696.81
	G	m ² ha ⁻¹	213	33.25	0.29	85.40	14.35
Douglas fir	H_{100}	m	345	20.23	6.89	42.02	7.62
	T	years	431	36.70	11	120	17.27
	D_g	cm	431	24.52	6.73	87.15	14.02
	N	ha ⁻¹	431	1096.79	8.51	5175.14	996.41
	G	m ² ha ⁻¹	431	27.37	0.63	62.66	9.30

Chapter 3

Dynamic, stand-level growth models

Stand-level growth models are models in which the basic units are stand-level variables such as basal area, stand volume, stem number, quadratic mean diameter, dominant height etc. Such models represent simplistic representations of the forest stand, but also generally require little information to predict forest growth. Thus, the general information produced by stand-level models are of great value in forest practice as necessary information can easily be obtained in the forest to make predictions of future yield, expected revenues, or expected demands for machine or man power, etc..

Static models of forest growth, such as yield tables or closed-form mathematical models, fail to recognize that forest stands are dynamic systems, subject to sudden changes caused by for example natural hazards or silvicultural interventions. When the actual thinning practice differs from that of the equation or table, the model can no longer be expected to provide sensible predictions. Changing market conditions, variations in management objectives, and natural hazards during the lifespan of the stand frequently alter the applied silvicultural practice. Therefore, the importance of developing dynamic models for forest management planning is evident.

In VIDAR the main component are the dynamic stand-level growth models previously estimated for beech (Nord-Larsen, 2006b), Norway spruce (Leary et al., 1999; Skovsgaard et al., 1999; Johannsen et al., 1999), oak (Johannsen, 1999), and Douglas fir (Christensen et al., 2003). Additionally, similar models have been estimated for Sitka spruce and European silver fir during the development of VIDAR. The dynamic aspect of the models is the main feature distinguishing this software from other forest management tools as it allows the user maximum flexibility without the need for prior assumptions often needed when using a static growth model (such as a yield table) in a dynamic planning tool. The models have been developed over a number of years and collection of

new data since the development of the individual models and demands for uniformity of the model structures have required that most of the models have been re-estimated for application in VIDAR.

3.1 Estimating dynamic stand-level growth models

The dynamic models in VIDAR are based on the state-space approach. The hypothesis underlying this approach is that the current state of the system reflects the past causes of the system and constitutes all the information needed to predict its future behavior. For instance, using this approach an even-aged beech stand may be characterized by the current basal area and dominant height and predictions of future basal area growth does not depend on the timing and intensity of thinnings leading to the current state of the system.

In the state-space model it is assumed that the n dimensional state vector at some point in time, $\mathbf{x}(t)$, can be predicted by the transition function \mathbf{F} of the state variable, $\mathbf{x}(t_0)$, and a vector of input variables, \mathbf{U} at some other point in time (after García, 1994):

$$\begin{aligned}\mathbf{y}(t) &= \mathbf{g}[\mathbf{x}(t)] \\ \mathbf{x}(t) &= \mathbf{F}[\mathbf{x}(t_0), \mathbf{U}, t - t_0]\end{aligned}\tag{3.1}$$

where the current output, \mathbf{y}_t is a function \mathbf{g} of the current state. In other words, the state-space approach predicts any future states of the system from the initial state, $\mathbf{x}(t_0)$, through iteration. For example, an initial observation of the two-dimensional state vector of height and basal area may be used to predict basal area and height after one period. The new estimates of the two state variables are then re-entered into the model to predict the state after one more period, and so forth. Abrupt changes in for example basal area due to thinnings are handled by simulating the shifts in the state vectors and are seen as shifts between different growth paths.

3.1.1 Stand-level model

The general model form for all species includes three equations for predicting dominant height growth (ΔH_{100}), basal area growth (ΔG), and mortality (ΔN):

$$\frac{\Delta H_{100,ij}}{\Delta t} = a_j H_{100,ij}^{\alpha_1} e^{\alpha_2 H_{100,ij} + \alpha_3 G_{ij}} + \varepsilon_{H,ij} \quad (3.2.1)$$

$$\frac{\Delta G_{ij}}{\Delta t} = (\beta_{01} + \beta_{02} \cdot a_j) G_{ij}^{\beta_1} e^{\beta_2 G_{ij} + \beta_3 H_{100,ij}^{\beta_4}} + FV[G]_{i+1,j} + \varepsilon_{G,ij} \quad (3.2.2)$$

$$\frac{\Delta N_{ij}}{\Delta t} = -\gamma_1 N_{ij}^{\gamma_2} e^{\gamma_3 \sqrt{N_{ij}} H_{100,ij}} + FV[N]_{i+1,j} + \varepsilon_{N,ij} \quad (3.2.3)$$

where a_j is a site-specific parameter estimated locally for the j th plot. The remaining parameters, $\alpha_1 - \alpha_3$, $\beta_{01} - \beta_4$, and $\gamma_1 - \gamma_3$, are estimated globally. $\varepsilon_{H,ij} \sim N(0, \sigma_H^2)$ and $\varepsilon_{G,ij} \sim N(0, \sigma_G^2)$ are the error terms of the i th measurement occasion on the j th plot. $FV[G]$ and $FV[N]$ (Forcing Value) represents the shift in G and N caused by thinning.

For practical application of the stand model, a_j must be estimated from a series of observations of height and basal area. When the model is applied where beech has not been grown before or when there are no sequential observations of stand variables the estimation cannot be carried out. A preliminary study showed that a was highly correlated with the more traditional measure of site quality, site index, defined as the dominant height at age 50. To allow flexible use of the model, depending on the available data, we also estimated the stand level model where site specific effects were substituted by a linear function of site index (S):

$$\frac{\Delta H_{100,ij}}{\Delta t} = (\alpha_{01} + \alpha_{02} \cdot S_j) H_{100,ij}^{\alpha_1} e^{\alpha_2 H_{100,ij} + \alpha_3 G_{ij}} + \varepsilon_{H,ij} \quad (3.3.1)$$

$$\frac{\Delta G_{ij}}{\Delta t} = (\beta_{01} + \beta_{02} \cdot S_j) G_{ij}^{\beta_1} e^{\beta_2 G_{ij} + \beta_3 H_{100,ij}^{\beta_4}} + FV[G]_{i+1,j} + \varepsilon_{G,ij} \quad (3.3.2)$$

$$\frac{\Delta N_{ij}}{\Delta t} = -\gamma_1 N_{ij}^{\gamma_2} e^{\gamma_3 \sqrt{N_{ij}} H_{100,ij}} + FV[N]_{i+1,j} + \varepsilon_{N,ij} \quad (3.3.3)$$

The three equations in the stand-level models were estimated following the approach of McDill and Amateis (1993). Following this approach, the estimation problem may be written as a series of annual difference equations that increment stand height, stand basal area or stem numbers from some initial state to the state at some later point in time, using the years between the two observations

as the number of iterations. Considering height, the state at the end of the growth period may be predicted from the state at the beginning of the growth period by a series of predicted annual increments:

$$\hat{H}_{i+1,j} = H_{i,j} + f(H_{i,j}, G_{i,j}) + \varepsilon_{i+1,j} \quad (3.4.1)$$

$$\hat{H}_{i+2,j} = \hat{H}_{i+1,j} + f(\hat{H}_{i+1,j}, \hat{G}_{i+1,j}) + \varepsilon_{i+2,j} \quad (3.4.2)$$

\vdots

$$\hat{H}_{i+t,j} = \hat{H}_{i+t-1,j} + f(\hat{H}_{i+t-1,j}, \hat{G}_{i+t-1,j}) + \varepsilon_{i+t,j}, \quad (3.4.j)$$

where $f(H_{i,j}, G_{i,j})$ is expressed in equation (3.2.1) and models annual height increment of the j th plot at time $i + t$ ($t = 0, 1, 2, \dots, n$). The parameters of the annual difference equation were then estimated using a nonlinear least squares procedure (Marquardt in PROC MODEL, SAS 8.2) that minimized the squared deviations of $\hat{H}_{i+t,j}$ from $H_{i+t,j}$.

The data used for this study represents a structure of repeated measurements on individual plots. Failure to recognize that within-plot measurements are correlated may result in inefficient estimates and underestimated standard errors when correlations are strong. When growth is viewed as an incremental process where only current conditions influence current growth, the problems of serial correlation are usually avoided (García, 1983; Seber and Wild, 1989). However, we explicitly modeled the serial correlation by including a generalized formulation of the first-order autoregressive model that accommodates the irregular spacing of measurements:

$$\varepsilon_{ij} = \rho_m^{t_{ij}-t_{i-1,j}} \varepsilon_{i-1,j} + u_{ij} \quad (i = 1, 2, \dots), \quad (3.5)$$

where ε_i is the error at the i th measurement, t is the time, ρ_m is the coefficient of correlation of the m th equation and the u_i 's are normally and independently distributed random errors.

3.2 Results

3.2.1 Beech

Parameter estimates of the system of equations (3.2.1) – (3.2.3) were all significant ($P < 0.05$) except for β_{01} that was eliminated from the model. After reduction of the models all parameters were significant. The correlation coefficient of the height model (ρ_H in equation (3.5)) was non-significant,

indicating no correlation of height growth in subsequent growth periods. The correlation coefficient of basal area growth (ρ_G) was highly significant, which may indicate that basal area growth in subsequent periods was positively correlated or may be a consequence of model misspecification.

The reduced model system, using the site specific parameter a (equation (3.2.1) – (3.2.3)) accounted for more than 98% of the observed variation of H_{100} , G and N at the end of the growth period (Table 3.1). Based on periodic annual increment (PAI), the height and basal area models explained 33% and 72% of the total variation in annual growth, respectively, whereas the mortality model explained 44% of the observed annual change in stem numbers. The relatively low coefficient of explanation of PAI is probably due to short measurement intervals, which decreases the signal-to-noise ratio.

Parameter estimates of the model system using site index as the site specific variable (3.3.1) – (3.3.3) were all significant ($P < 0.05$) except for α_{01} and β_{01} that were both eliminated from the model. The use of site index as the site-specific variable resulted in only a slight decrease in precision (Table 3.1).

Residuals of the stand growth model were evenly distributed around zero and revealed no apparent irregularities when plotted against the predicted variables (Figure 3.1).

3.2.2 Oak

Parameter estimates of the system of equations (3.2.1) – (3.2.3) were all significant ($P < 0.05$) except for α_2 . However, this parameter determines the shape of the growth path and was consequently preserved in the final model. The correlation coefficient of the height growth model (ρ_H) was not significant indicating that height growth in subsequent periods were not correlated. Conversely, the correlation coefficient of the basal area growth model (ρ_G) was highly significant, which may indicate that basal area growth in subsequent periods is positively correlated or may be a consequence of model misspecification.

The reduced model system, using the site specific parameter a (equation (3.2.1) – (3.2.3)) accounted for about 98% of the observed variation of H_{100} , G and N at the end of the growth period (Table 3.2). Based on PAI the height and basal area models explained 42% and 76% of the total variation in annual growth, respectively, whereas the mortality model explained 37% of the observed annual changes in stem numbers.

Parameter estimates of the model system using site index as the site specific variable (3.3.1) – (3.3.3) were all significant ($P < 0.05$) except for α_{01} . The use of site index as the site specific variable resulted in only a slight decrease in precision (Table 3.2).

Table 3.1: Parameter estimates for beech of the system of equations presented in equations (3.2.1) – (3.2.3) and (3.3.1) – (3.3.3) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.0235 ^a	0.0074 ^a	0.9914	.	.	0.9913
	α_{02}	.	.		2.089E-3	5.87E-4	
	α_1	2.0325	0.1659		1.7623	0.1610	
	α_2	-0.1854	0.0104		-0.1764	0.0101	
	α_3	0.0117	2.03E-3		0.0151	1.96E-3	
	ρ_H	0.0206	0.1955		0.0247	0.1997	
G	β_{02}	28.96437	8.7957	0.9912	0.0457	6.49E-3	0.9885
	β_1	0.5494	0.0625		0.6510	0.0664	
	β_2	-0.0151	2.72E-3		-0.0166	3.15E-3	
	β_3	-0.0381	0.0130		-0.1616	0.0451	
	β_4	1.1086	0.0886		0.7705	0.0671	
	ρ_G	0.6246	0.0244		0.7555	0.0143	
N	γ_1	1.1125E-3	1.82E-4	0.9871	1.0264E-3	1.70E-4	0.9873
	γ_2	1.	.		1.	.	
	γ_3	0.0311	1.40E-3		0.0317	1.40E-3	

^aEstimated individually for each experiment, number represents a simple average.

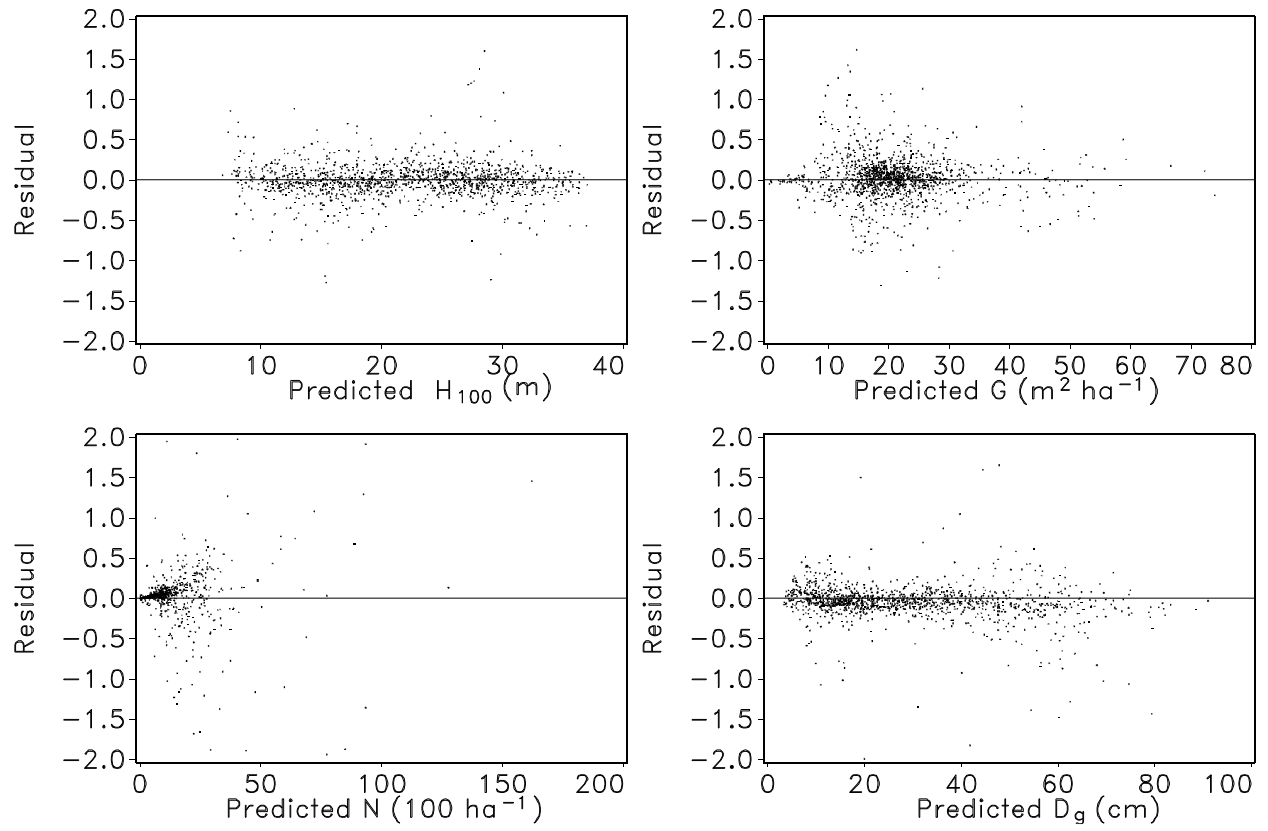


Figure 3.1: Residuals of the stand-level model for beech. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

Table 3.2: Parameter estimates for oak of the system of equations presented in equations (3.2.1) – (3.2.3) and (3.3.1) – (3.3.3) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.7579 ^a	0.0100 ^a	0.9883	.	.	0.9880
	α_{02}	.	.		3.3847E-3	1.24E-3	
	α_1	1.5422	0.2198		1.7565	0.2221	
	α_2	-0.1942	0.0152		-0.2105	0.0151	
	α_3	4.8474E-3	3.35E-3		6.9736E-3	3.26E-3	
	ρ_H	9.8799E-3	0.2381		9.4343E-3	0.2406	
G	β_{02}	7.3517	2.8938	0.9848	0.02747	6.17E-3	0.9826
	β_1	1.0065	0.1236		1.0144	0.1243	
	β_2	-0.0549	7.12E-3		-0.0524	7.20E-3	
	β_3	-0.1984	0.0592		-0.1204	0.0342	
	β_4	0.7926	0.0754		0.9290	0.0736	
	ρ_G	0.4565	0.0559		0.6072	0.0323	
N	γ_1	7.3332E-5	3.60E-5	0.9798	8.7927E-5	4.30E-5	0.9801
	γ_2	1.	.		1.	.	
	γ_3	0.0568	4.14E-3		0.0555	4.14E-3	

^aEstimated individually for each experiment, number represents a simple average.

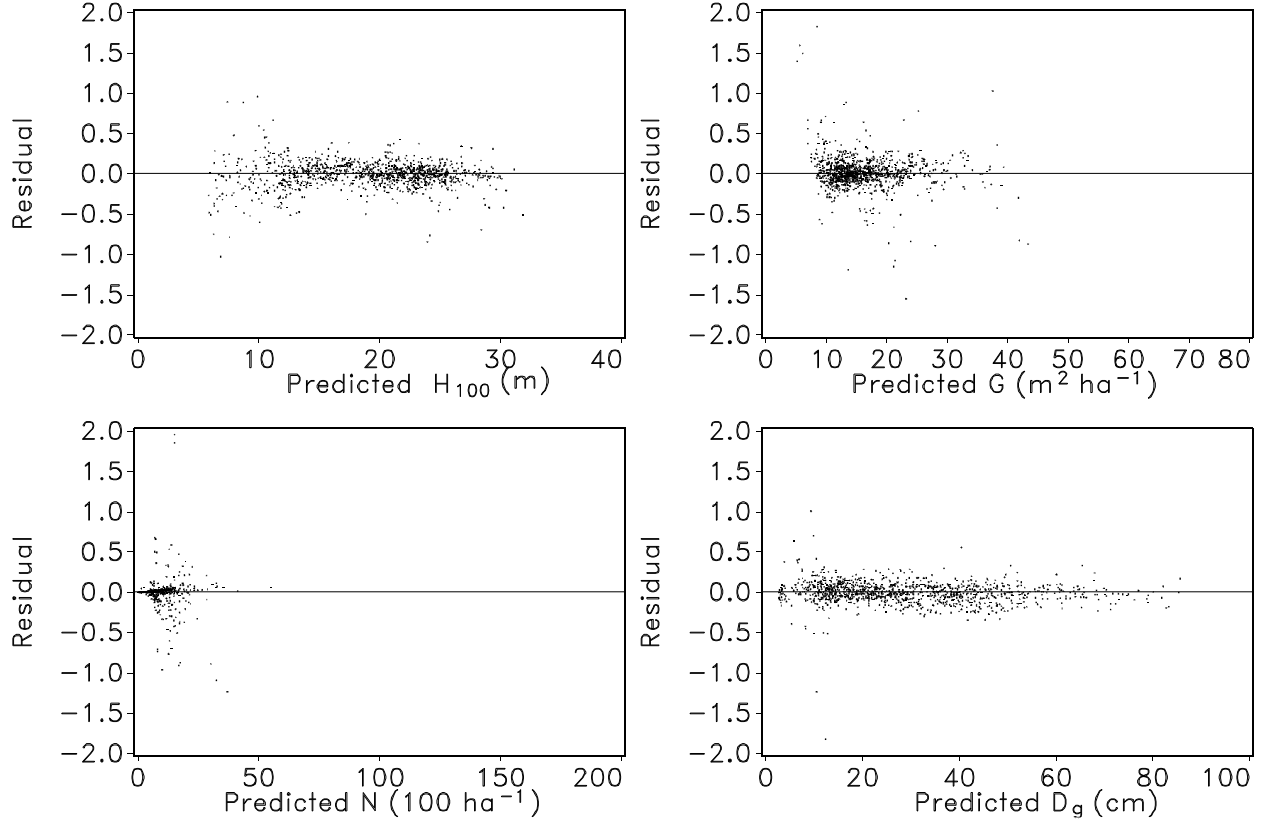


Figure 3.2: Residuals of the stand-level model for oak. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

Residuals of the stand growth model for oak were evenly distributed around zero and revealed no apparent irregularities when plotted against the predicted variables (Figure 3.2).

3.2.3 Norway spruce

Parameter estimates of the system of equations (3.2.1) – (3.2.3) were all significant ($P < 0.05$) except for β_{01} and β_3 . The former was eliminated from the model, whereas the latter was considered necessary for the model and was consequently not left out. After reduction of the models all parameters were significant. The correlation coefficients of height (ρ_H) and basal area growth models (ρ_G) were highly significant, which may indicate that height and basal area growth in subsequent

periods was positively correlated or may be a consequence of model misspecification.

The reduced model system, using the site specific parameter a (equation (3.2.1) – (3.2.3)) accounted for more than 97% of the observed variation of H_{100} , G and N at the end of the growth period (Table 3.3). Based on PAI the height and basal area models explained 28% and 60% of the total variation in annual growth, respectively, whereas the mortality model explained 23% of the observed annual change in stem numbers.

Parameter estimates of the model system using site index as the site specific variable (3.3.1) – (3.3.3) were all significant ($P < 0.05$). The use of site index as the site specific variable resulted in only a slight decrease in precision (Table 3.3).

Residuals of the stand growth model of height and basal area for Norway spruce were evenly distributed around zero and revealed no apparent irregularities when plotted against the predicted variables (Figure 3.3). However, predictions of stem numbers were clearly biased leading also to biased estimates of quadratic mean diameter.

3.2.4 Sitka spruce

Many of the parameter estimates of the system of equations (3.2.1) – (3.2.3) not significant (including most of the site-specific parameters) ($P \geq 0.05$). The reason for the non-significant parameters is probably that the signal-to-noise ratio is relatively large due to short time spans between measurements. However, to maintain the general model structure we chose not to eliminate non-significant parameters. The correlation coefficients of height (ρ_H) and basal area growth models (ρ_G) were both significant. This may indicate that height growth in subsequent periods is negatively correlated, but more likely it is an effect of measurement errors. Basal area growth on the other hand is probably positively correlated in subsequent periods.

The model system, using the site specific parameter a (equation (3.2.1) – (3.2.3)) accounted for more than 98% of the observed variation of H_{100} , G and N at the end of the growth period (Table 3.4). Based on PAI the height and basal area models explained 10% and 41% of the total variation in annual growth, respectively.

As with the site specific parameter model, many parameter estimates of the model system using site index as the site specific variable (3.3.1) – (3.3.3) were non-significant ($P \geq 0.05$). The use of site index as the site specific variable resulted in only a slight decrease in precision (Table 3.4).

Residuals of the stand growth model for Sitka spruce were evenly distributed around zero and revealed no apparent tendencies across the predicted variables (Figure 3.4).

Table 3.3: Parameter estimates for Norway spruce of the system of equations presented in equations (3.2.1) – (3.2.3) and (3.3.1) – (3.3.3) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.0064 ^a	0.0019 ^a	0.9790	.	.	0.9785
	α_{01}	.	.		-1.7229E-3	5.60E-4	
	α_{02}	.	.		4.0060E-4	1.24E-4	
	α_1	2.9138	0.1971		3.0205	0.1866	
	α_2	-0.2458	0.0132		-0.2541	0.0126	
	α_3	-2.6906E-3	1.37E-3		-6.4661E-4	1.23E-3	
	ρ_H	-0.0555	0.0964		-0.3763	0.0535	
G	β_{01}	-0.0147	9.55E-3	0.9690	-0.0513	0.0164	0.9631
	β_{02}	35.0689	13.5287		0.0135	3.82E-3	
	β_1	1.0608	0.1247		1.1248	0.1308	
	β_2	-0.0437	4.36E-3		-0.0429	4.74E-3	
	β_3	-0.0165	0.0135		-0.0185	0.0150	
	β_4	1.2827	0.2281		1.2595	0.2255	
	ρ_G	0.5644	0.0321		0.7389	0.0144	
N	γ_1	2.6697E-3	3.20E-4	0.9837	2.7832E-3	3.26E-4	0.9838
	γ_2	1.	.		1.	.	
	γ_3	0.0182	1.28E-3		0.0176	1.24E-3	

^aEstimated individually for each experiment, number represents a simple average.

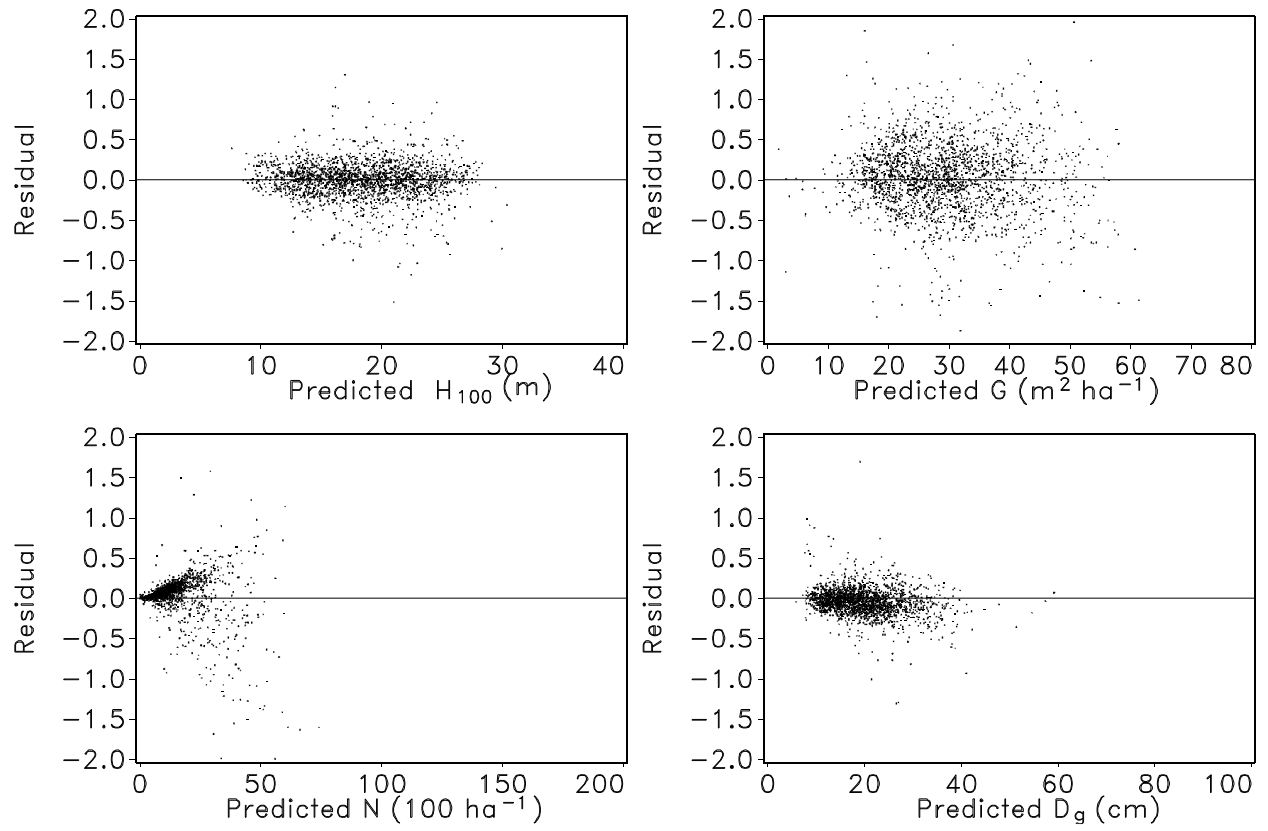


Figure 3.3: Residuals of the stand-level model for Norway spruce. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

Table 3.4: Parameter estimates for Sitka spruce of the system of equations presented in equations (3.2.1) – (3.2.3) and (3.3.1) – (3.3.3) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.0289 ^a	0.0210 ^a	0.9825	.	.	0.9815
	α_{01}	.	.		4.8390E-3	5.31E-3	
	α_{02}	.	.		1.0496E-3	8.09E-4	
	α_1	2.1681	0.4210		2.0415	0.4234	
	α_2	-0.1622	0.0249		-0.1615	0.0249	
	α_3	-3.9901E-3	2.23E-3		-1.3841E-3	2.12E-3	
	ρ_H	-0.2369	0.0877		-0.2424	0.0864	
G	β_{01}	-0.1244	0.0872	0.9809	-0.0411	0.0340	0.9741
	β_{02}	13.1947	12.5240		7.2996E-3	5.13E-3	
	β_1	1.1845	0.2566		1.4101	0.2951	
	β_2	-0.0344	6.96E-3		-0.0395	8.66E-3	
	β_3	-9.9601E-3	0.0108		-0.0290	0.0281	
	β_4	1.4340	0.2975		1.2075	0.2588	
	ρ_G	0.3311	0.0770		0.7895	0.0233	
N	γ_1	2.0197E-4	1.82E-4	0.9963	1.4612E-4	1.26E-4	0.9964
	γ_2	1.	.		1.	.	
	γ_3	0.0376	0.0102		0.0422	9.66E-3	

^aEstimated individually for each experiment, number represents a simple average.

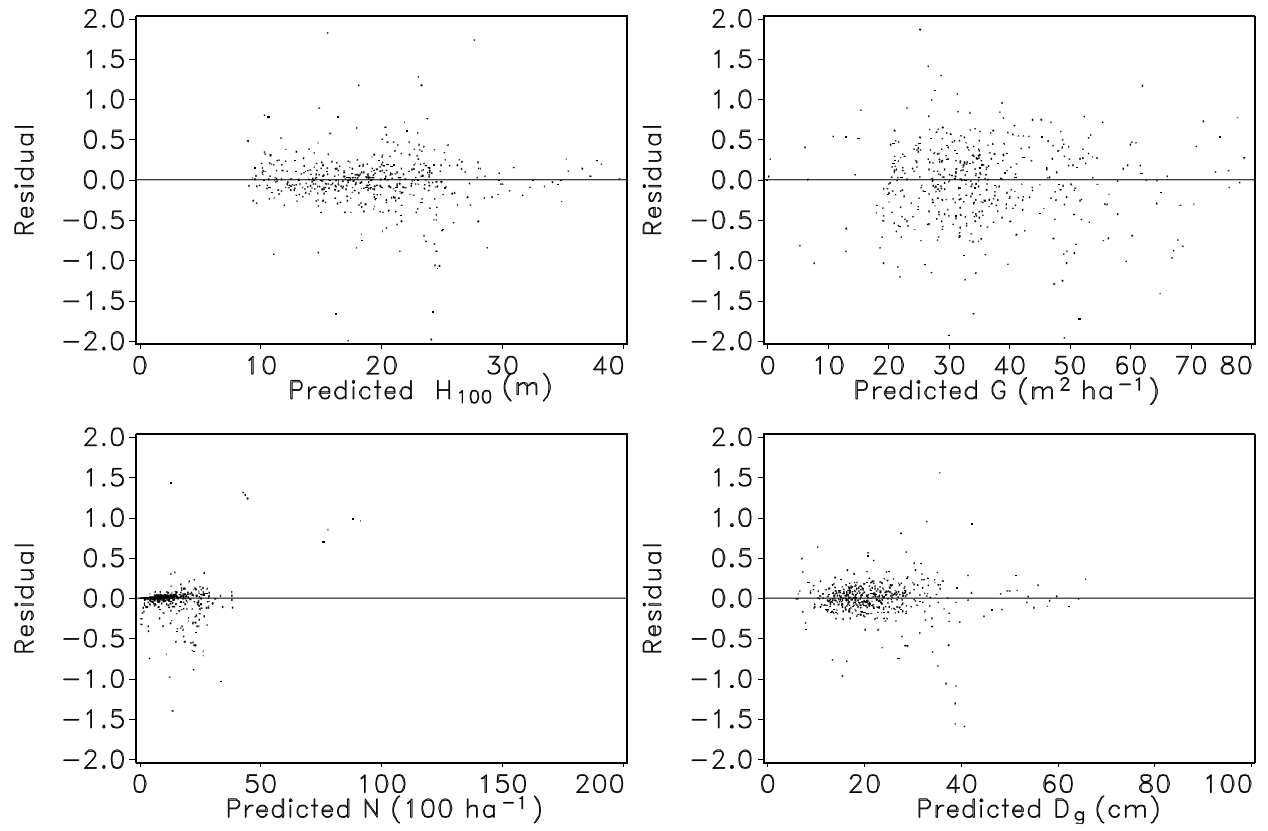


Figure 3.4: Residuals of the stand-level model for Sitka spruce. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

3.2.5 Silver fir

As for Sitka spruce, many of the parameter estimates of the system of equations (3.2.1) – (3.2.3) not significant (including most of the site-specific parameters) ($P \geq 0.05$). Similarly to the models for Sitka spruce we chose to maintain the general model structure and hence not to eliminate non-significant parameters. The correlation coefficient for height (ρ_H) was non-significant, indicating that height growth in subsequent periods was uncorrelated. Oppositely, the correlation coefficient of the basal area growth model (ρ_G) was significant which may indicate that basal area growth in subsequent periods is positively correlated.

The model system, using the site specific parameter a (equation (3.2.1) – (3.2.3)) accounted for more than 98% of the observed variation of H_{100} , G and N at the end of the growth period (Table 3.5). Based on PAI the height and basal area models explained 5% and 70% of the total variation in annual growth, respectively. The small coefficient of explanation for the height growth model is probably due to short time-span between measurements and the uncertainty related to height measurements in general which causes a low signal-to-noise ratio.

As with the site specific parameter model, many parameter estimates of the model system using site index as the site specific variable (3.3.1) – (3.3.3) were non-significant ($P \geq 0.05$). The use of site index as the site specific variable resulted in only a slight decrease in precision (Table 3.5).

Residuals of the stand growth model for silver fir were evenly distributed around zero and revealed no apparent tendencies across the predicted variables (Figure 3.5).

3.2.6 Douglas fir

Estimation of the stand model for Douglas fir deviated from estimating the model for the other tree species regarding the number of observations available for estimation. Also the range of site conditions and treatments were more limited. This affected the parameter estimation in several ways. Firstly, it was not possible to obtain convergence when including the mortality model. This is possibly due to the lack of unthinned or lightly thinned stands. Hence, no mortality model was estimated for Douglas fir. Secondly, although it was possible to obtain convergence using the remaining models and parameters, many of the parameters were not significant. This led us to fix the α_1 and β_1 parameters in equations (3.2.1) and (3.2.2) at 1 in accordance with the original formulation of the model (Christensen et al., 2003).

After the reduction of the system of equations described above all parameter estimates of the height growth model (3.2.1) were significant. In the basal area growth model (3.2.2), parameters β_{01}

Table 3.5: Parameter estimates for silver fir of the system of equations presented in equations (3.2.1) – (3.2.3) and (3.3.1) – (3.3.3) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.0225 ^a	0.0207 ^a	0.9896	.	.	0.9896
	α_{01}	.	.		-3.1727E-3	4.64E-3	
	α_{02}	.	.		2.5511E-3	2.28E-3	
	α_1	2.0251	0.5821		1.7164	0.5064	
	α_2	-0.1607	0.0334		-0.1493	0.0276	
	α_3	3.3279E-4	5.07E-3		2.4291E-4	4.08E-3	
	ρ_H	.	.		-0.2022	0.5083	
G	β_{01}	4.7081E-3	0.1229	0.9828	5.7007E-3	0.0836	0.9784
	β_{02}	62.5248	77.5825		0.0689	0.0465	
	β_1	0.1512	0.2990		0.3191	0.3181	
	β_2	4.9951E-3	9.20E-3		-2.3131E-3	9.06E-3	
	β_3	-2.0296E-3	3.11E-3		-8.2618E-3	0.0122	
	β_4	2.0072	0.4320		1.6346	0.4013	
	ρ_G	.	.		0.8672	0.0359	
N	γ_1	7.1174E-4	4.94E-4	0.9960	8.5411E-4	5.60E-4	0.9962
	γ_2	1.	.		1.	.	
	γ_3	0.0315	0.0111		0.0280	0.0105	

^aEstimated individually for each experiment, number represents a simple average.

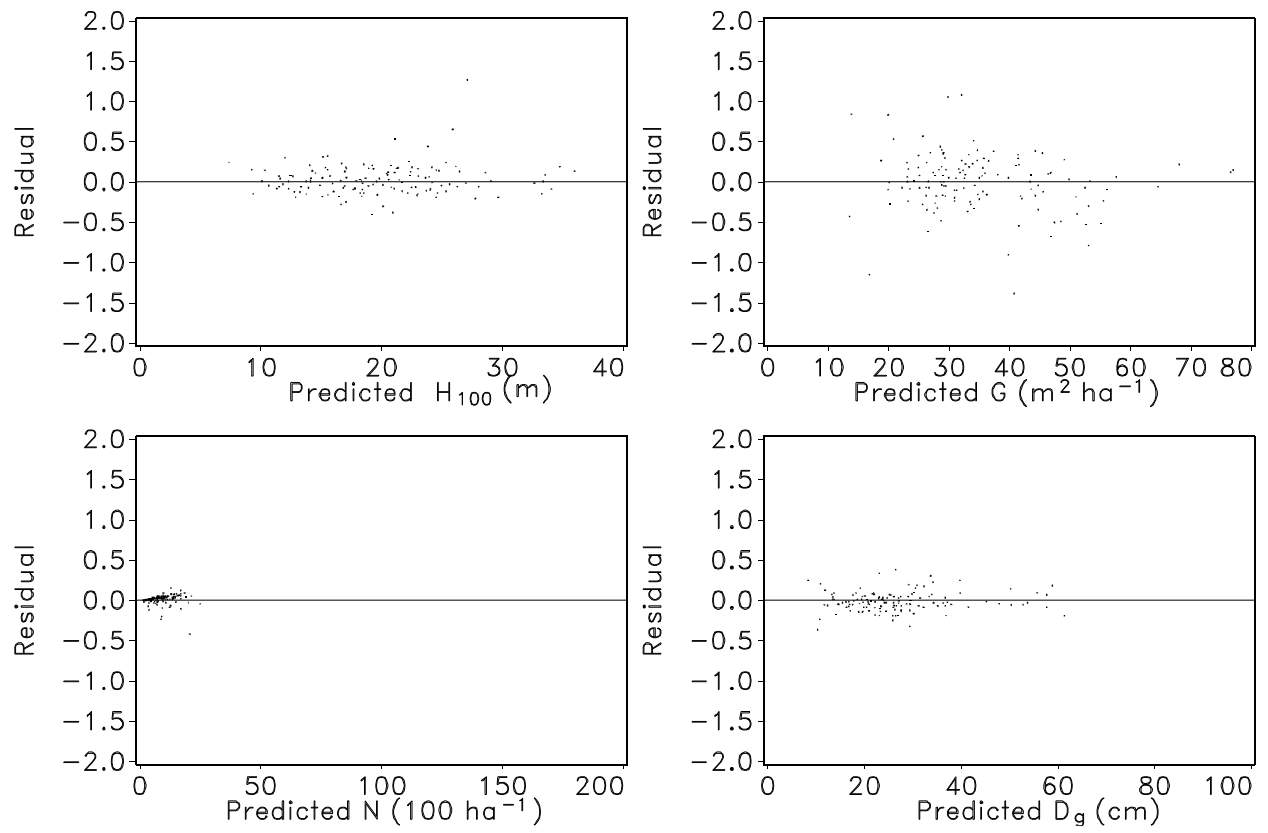


Figure 3.5: Residuals of the stand-level model for silver fir. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

and β_{02} were insignificant ($P > 0.05$), which suggests no site specific effects on basal area growth. This is possibly due to the limited variation in growth conditions reflected in the data, and should not be interpreted as a general feature of Douglas fir in Denmark. When the model was estimated after removing β_{01} , β_{02} became significant, but this resulted in a 10% increase in RMSE. Hence, we used the original model formulation, accepting that some parameters were insignificant as a result of the limited number of observations.

The correlation coefficient of the height growth model (ρ_H) was not significant indicating that height growth in subsequent periods were not correlated. Conversely, the correlation coefficient of the basal area growth model (ρ_G) was highly significant. This may indicate that basal area growth in subsequent periods is positively correlated or may be a consequence of model misspecification.

The reduced model system, using the site-specific parameter a (equation (3.2.1) – (3.2.3)) accounted for about 97% of the observed variation of H_{100} and G at the end of the growth period (Table 3.6).

Parameter estimates of the model system using site index as the site-specific variable (3.3.1) – (3.3.2) were all significant ($P < 0.05$) except for α_{01} . The use of site index as the site-specific variable resulted in only a slight decrease in precision (Table 3.6).

Table 3.6: Parameter estimates for Douglas fir of the system of equations presented in equations (3.2.1) – (3.2.2) and (3.3.1) – (3.3.2) along with their standard errors. R^2 was calculated from the deviations between predicted and observed values at the end of the growth periods.

Site parameter		a			Site index		
Model	Parameter	Estimate	Std. error	R^2	Estimate	Std. error	R^2
H_{100}	a	0.1674 ^a	0.0215 ^a	0.9881	.	.	0.9870
	α_{01}	.	.		0.0889	0.0700	
	α_{02}	.	.		2.4227E-3	2.00E-3	
	α_1	1.3102	0.3604		1.0194	0.3641	
	α_2	-0.1260	0.0201		-0.1141	0.0205	
	α_3	0.0119	4.29E-3		0.0164	4.21E-3	
	ρ_H	.	.		-0.03429	0.4960	
G	β_{01}	-7.5318E-4	0.0802	0.9823	1.2289	0.8830	0.9759
	β_{02}	4.0649	3.5554		0.0404	0.0300	
	β_1	0.7504	0.2538		0.03027	0.3031	
	β_2	-9.9993E-3	8.63E-3		0.0143	0.0104	
	β_3	-6.7715E-3	4.86E-3		-0.0126	9.67E-3	
	β_4	1.5605	0.1940		1.3966	0.2048	
	ρ_G	.	.		0.7028	0.0479	
N	γ_1	5.8784E-4	1.69E-3	0.9796	5.3639E-4	1.56E-3	0.9797
	γ_2	1.	.		1.	.	
	γ_3	0.0308	0.0467		0.0322	0.0470	

^aEstimated individually for each experiment, number represents a simple average.

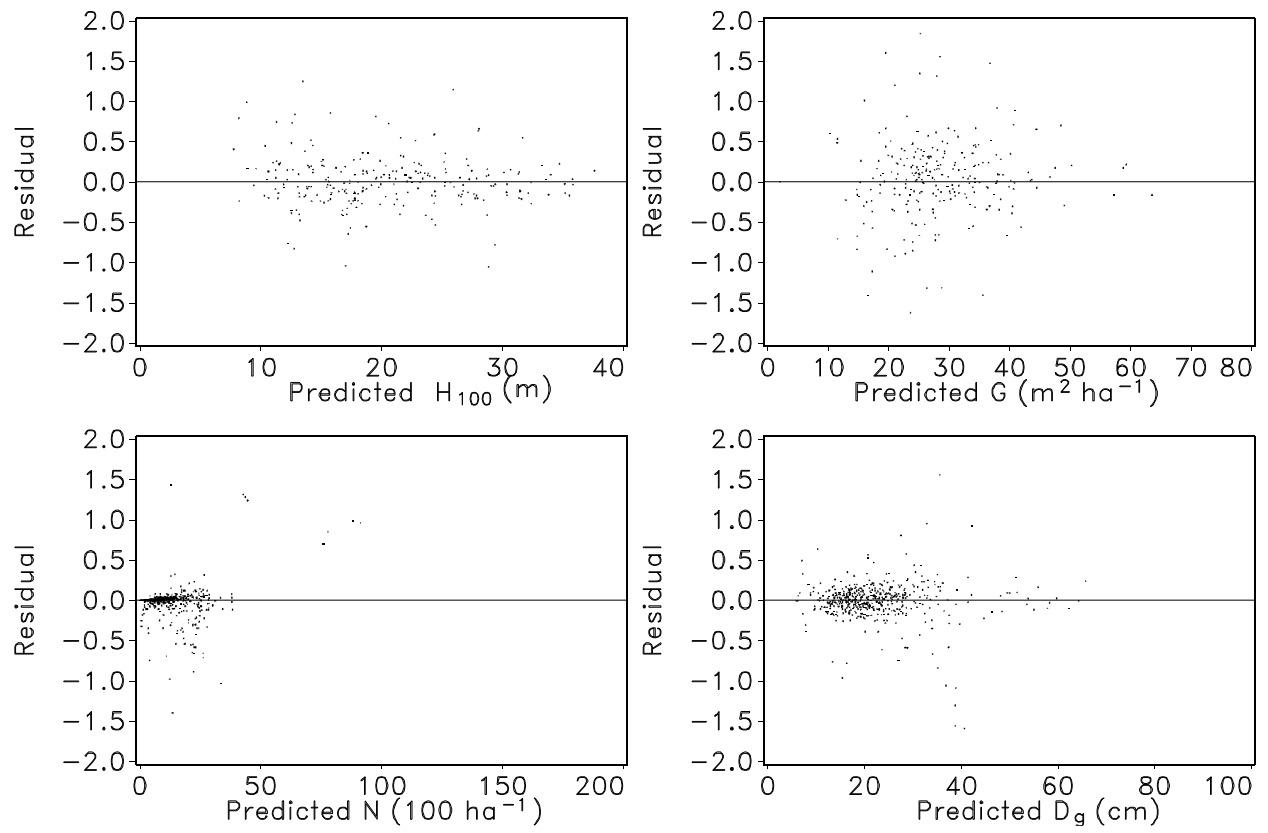


Figure 3.6: Residuals of the stand-level model for Douglas fir. Residuals are calculated as the difference between observed and predicted periodic annual growth (PAI).

Chapter 4

Site index models

Some assessment of site quality is required for making meaningful forecasts of forest growth and yield and a vast number of studies have been devoted to the characterization of site quality. One of the most common methods for assessing site quality in forest practice is to relate a specific height-age observation to a standardized set of age-height growth patterns called site curves (Danish: "bonitetskurver"). The expected height (site index) at a specific age (base-age) expresses the site quality.

Site index models are mathematical expressions of the expected height-age development given a specific height-age observation. Such models are crucial in VIDAR as they are used in the dynamic models to express differences in site quality. Site index models were developed in relation to the development of dynamic models for beech (Nord-Larsen, 2006a) but had not previously been developed for the other tree species in VIDAR. This chapter documents the development of site index models for beech, Norway spruce, oak, and Douglas fir.

4.1 Developing base-age invariant site index models

Apart from early attempts with graphical smoothing of the height-age relationship, most recent work on site index curves involves fitting of functions of the general form $H = f(t, S, \boldsymbol{\beta})$ where t is some age, S is the height at base age and $\boldsymbol{\beta}$ is a vector of parameters. These functions predict height as a function of age and site index. Such functions are often called static or base-age specific models, because they predict height as a function of age and a site index at a fixed base-age obtained from a direct height measurement.

When fitting base-age specific site curves to repeated measurements on permanent sample plots, assessment of base-age and site index may be a problem because such data rarely have height measurements at an age common to all plots or at the desired base age. Hence, heights at the desired base age must be obtained from inter- or even extrapolations for all plots not measured at the chosen base age. A different type of models unequivocally defines the individual site curves from any age-height point on the curve. Such base-age invariant models evade the need for prior assessment of site index as height at base age is expressed explicitly by any height-age observation on the site curve. Hence, site curves may be estimated from permanent sample plot data without any inter- or extrapolations to obtain site index.

Base-age invariant methods fit the three-dimensional surface (age, height and site index) in a manner that directly estimates the site-specific effects as parameters from the fitting procedure. Development of base-age invariant equations was first done by Bailey and Clutter (1974) using the *algebraic difference approach* (*ADA*). The method was subsequently used in a number of studies (e.g. Pienaar and Shiver, 1980; Amateis and Burkhart, 1985; Lenhart et al., 1986; Furnival et al., 1990) and was recently extended to a generalized form (*GADA*) by Cieszewski (2001, 2003); Cieszewski and Bailey (2000).

The development of site index models using *GADA* for beech is described by Nord-Larsen (2006a,b) and includes the following steps:

1. Select the basic equation form. This step is often performed using base-age specific methods to evaluate how well the equation represents the data.
2. Identify the site-specific parameter in the basic equation. Although Bailey and Clutter (1974) in their original formulation of the *ADA* describe models where site index depends solely on one parameter, any number may be specified in the *GADA*.
3. Generalize the site-specific parameters in the equation as functions of X , where X is an unknown measure of site quality. This step separates the *GADA* from *ADA* since introducing a function for the site-specific parameter allows it to vary across sites.
4. Solve for X and substitute for the independent variables, t and H , the initial conditions, t_0 and H_0 . The X in the equation developed in step 3 is finally replaced by this solution for X .

Among the models tested the best fit and most desirable properties (according to the list of nine characteristics presented by Goelz and Burk (1992)) was obtained by the three-parameter *GADA* age-height growth model derived by Cieszewski (2001, Equation (21)):

$$H = H_0 \frac{t^{\beta_1}(t_0^{\beta_1}R + \beta_2)}{t_0^{\beta_1}(t^{\beta_1}R + \beta_2)} \quad (4.1)$$

where:

$$R = Z_0 + \left(Z_0^2 + \frac{2\beta_2 H_0}{t_0^{\beta_1}} \right)^{0.5}$$

$$Z_0 = H_0 - \beta_3$$

where H_0 is the observed stand height at age t_0 and H is the estimated stand height at age t . β_1 , β_2 , and β_3 are model parameters. The equation is highly flexible and holds a number of desirable properties such as variable asymptotes and polymorphism. The derivation of this equation is not shown here and readers are referred to the original paper.

Equation (4.1) was solved using nonlinear least squares techniques. The site-specific parameter (H_0) was estimated locally for each experiment using an indicator variable method while the other parameters were simultaneously estimated globally. The properties of the model was assessed from a graphical analysis of curve shapes and residuals.

4.2 Results

4.2.1 Beech

Fitted site curves for beech generally represented the observed age-height patterns well across the entire range of site qualities (Figure 4.1). Parameter estimates are presented in Table 4.1. In the original paper, site curves for beech were estimated using both a base-age specific methodology and correcting for serial correlation. For VIDAR we used only the base-age specific estimation.

Fit statistics (Table 4.1) demonstrate that the site index equations represent the data well and that the model explains more than 99% of the the variation in dominant height. Residual plots (Figure 4.2) also show no severe trends across stand ages, predicted heights or for individual plots.

4.2.2 Oak

Compared to beech the age-height data for oak is somewhat less differentiated in terms of site index (Figure 4.3), although the observed range of site indices the site curves seem to represent the observed patterns well. For oak the parameter estimate of β_2 caused asymptotic height to decline

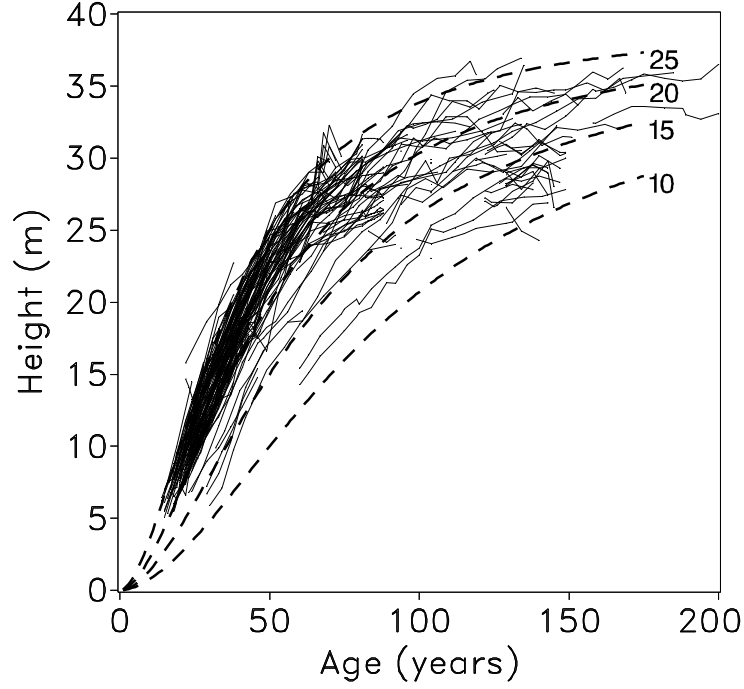


Figure 4.1: Site index curves for beech along with observed height-age patterns. Numbers refers to height at base age 50 years.

Table 4.1: Parameter estimates for the base-age invariant site equation for beech based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	1.7644	0.030	58.85	0.7367	0.991	0.501
	β_2	3472.7999	2196.2	1.71			
	β_3	36.8037	1.304	26.97			

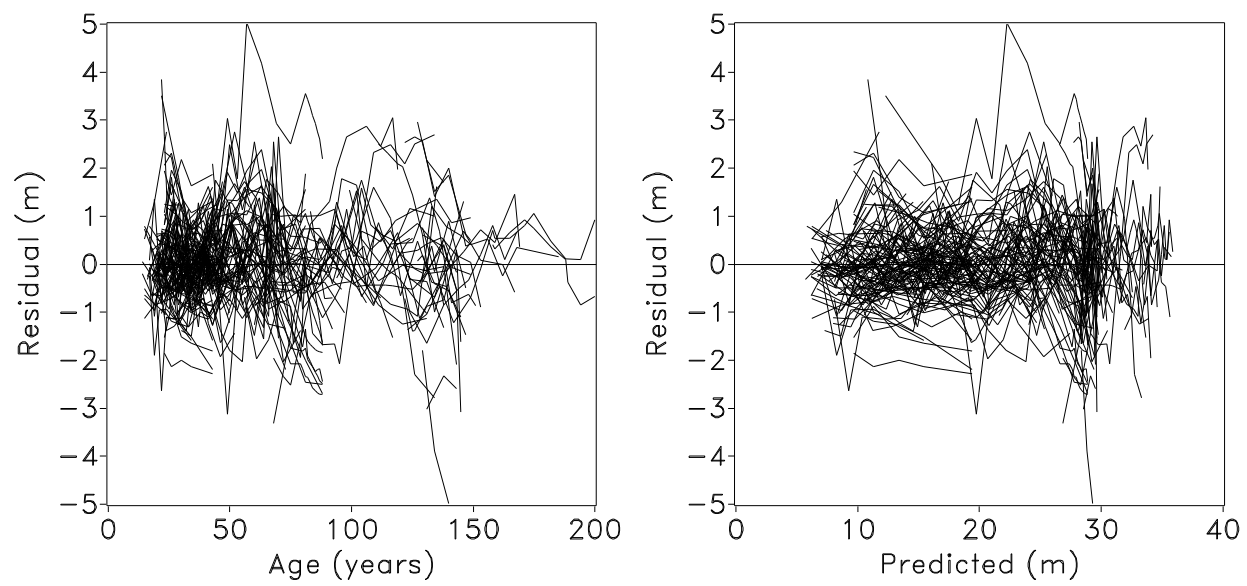


Figure 4.2: Residuals plotted against stand age and predicted dominant height for beech.

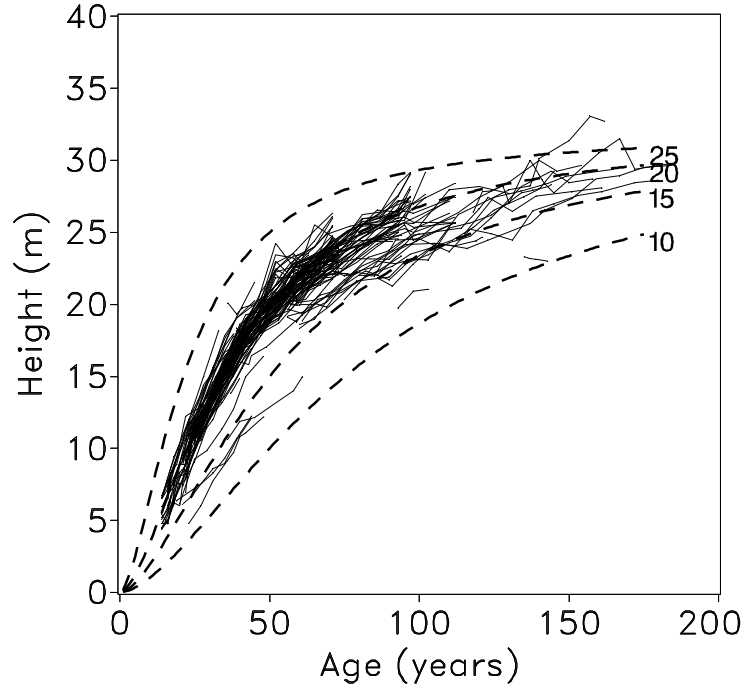


Figure 4.3: Site index curves for oak along with observed height-age patterns. Numbers refers to height at base age 50 years.

for high site index values and the model collapsed. In this case β_2 was arbitrarily restricted to 1, resulting in an increase in RMSE of 0.1 %. The restriction of β_2 causes the site curves to have identical asymptotes, but different shapes. Parameter estimates are presented in Table 4.2.

The site equations explain 99% of the variation of dominant height. The overall model fit is illustrated by the plots of residuals against stand age and predicted height (Figure 4.4). The plots show no apparent irregularities of the model across stand ages and predicted dominant heights or for individual plots.

4.2.3 Norway spruce

The site curves for Norway spruce generally represented the observed age-height patterns well across the entire range of site qualities (Figure 4.5). Parameter estimates are presented in Table 4.3.

Table 4.2: Parameter estimates for the base-age invariant site equation for oak based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	1.6277	0.0333	45.10	0.6158	0.990	0.410
	β_2	1.0000	.	.			
	β_3	31.9667	0.4187	45.10			

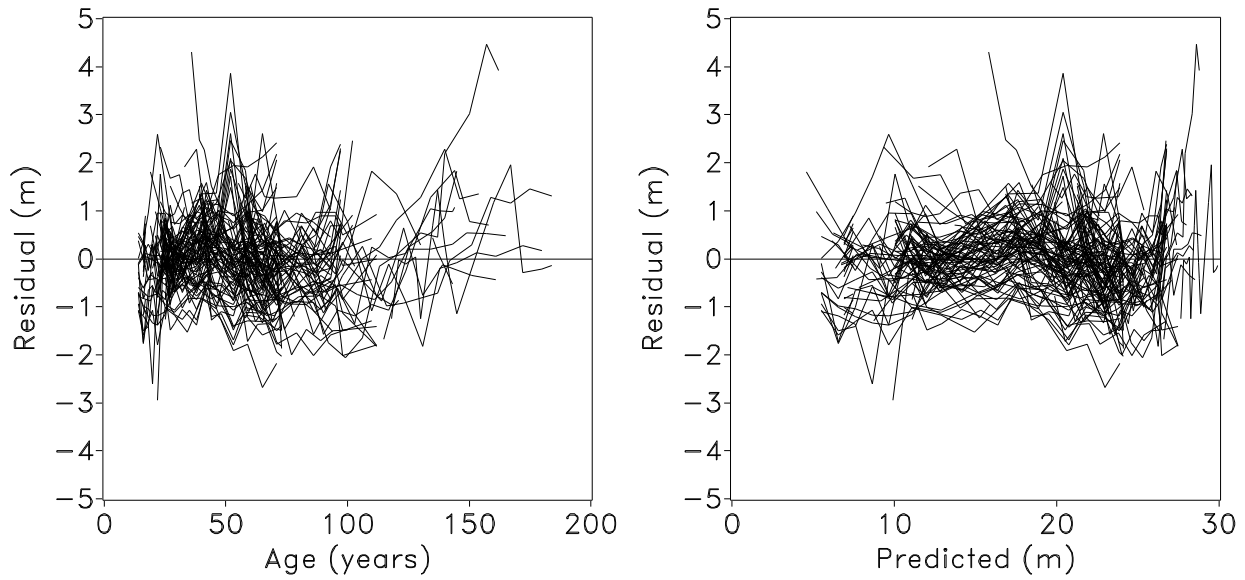


Figure 4.4: Residuals plotted against stand age and predicted dominant height for oak.

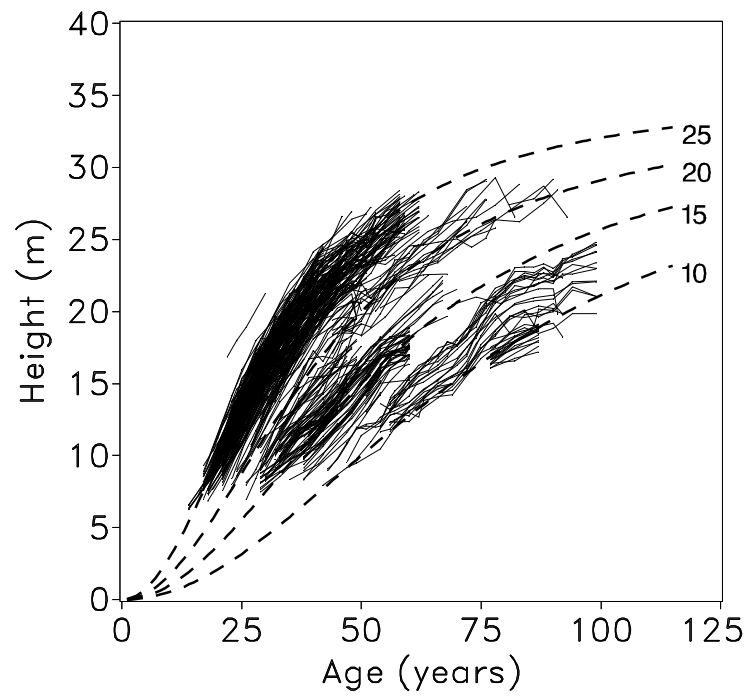


Figure 4.5: Site index curves for Norway spruce along with observed height-age patterns. Numbers refers to height at base age 50 years.

Table 4.3: Parameter estimates for the base-age invariant site equation for Norway spruce based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	2.0323	0.0303	65.55	0.6691	0.982	0.495
	β_2	6448.8721	2231.0	2.44			
	β_3	32.5165	0.9488	36.04			

Table 4.4: Parameter estimates for the base-age invariant site equation for Sitka spruce based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	2.1538	0.0872	24.02	0.807	0.984	0.547
	β_2	-2870.2666	7641.6	-0.41			
	β_3	38.8609	3.3895	11.60			

Fit statistics show that the model represents the data well, although it explains slightly less of the variation in dominant height ($\sim 98\%$) compared to the models for beech and oak ($\sim 99\%$). It seems that height growth at the good sites may be more persistent in old stands than predicted by the model, but no data is available to substantiate this finding. Nevertheless, the site equation seems to adequately predict dominant height growth within the common range of stand ages.

4.2.4 Sitka spruce

Sitka spruce shows a very rapid height growth compared to the other species. Nevertheless, the chosen site equation seems to represent the observed age-height pattern satisfactory (Figure 4.7). Parameter estimates are provided in Table 4.4.

The site equations for Sitka spruce explain 98% of the observed variation in dominant height and the residuals show no obvious trends across stand ages and predicted dominant heights (Figure 4.8).

4.2.5 Silver fir

The height growth pattern of silver fir is highly differentiated between good and poor sites. Nevertheless, the chosen site equation seems to represent the observed age-height pattern satisfactory (Figure 4.9). Parameter estimates are provided in Table 4.5.

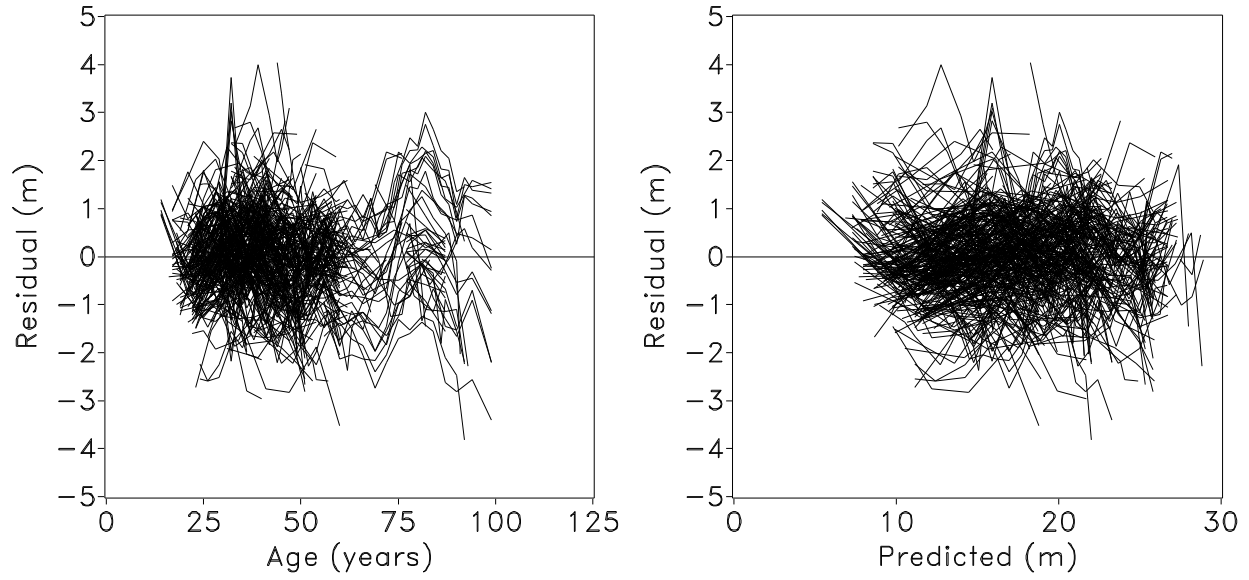


Figure 4.6: Residuals plotted against stand age and predicted dominant height for Norway spruce.

Table 4.5: Parameter estimates for the base-age invariant site equation for silver fir based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	2.1708	0.0740	28.78	0.7084	0.989	0.475
	β_2	140562.7517	40690.1	3.69			
	β_3	20.4927	4.1343	4.71			

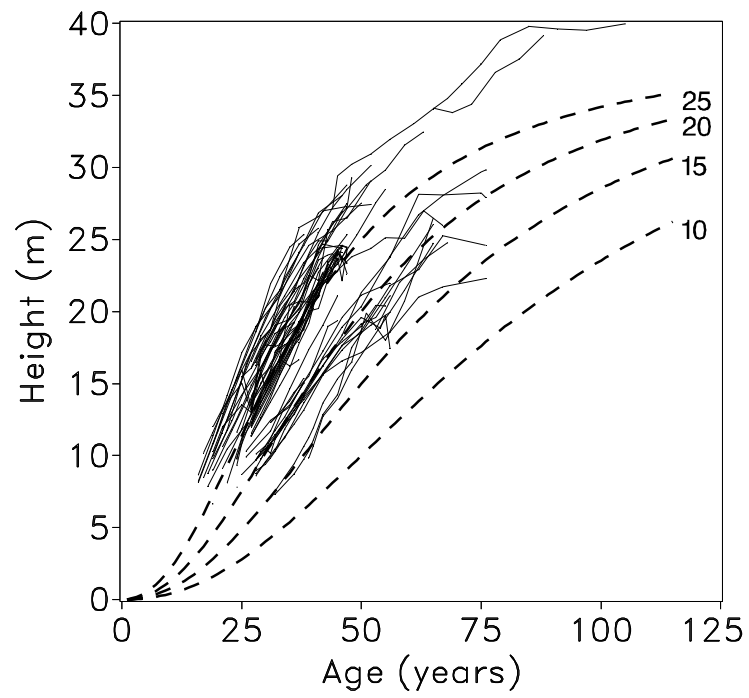


Figure 4.7: Site index curves for Sitka spruce along with observed height-age patterns. Numbers refers to height at base age 50 years.

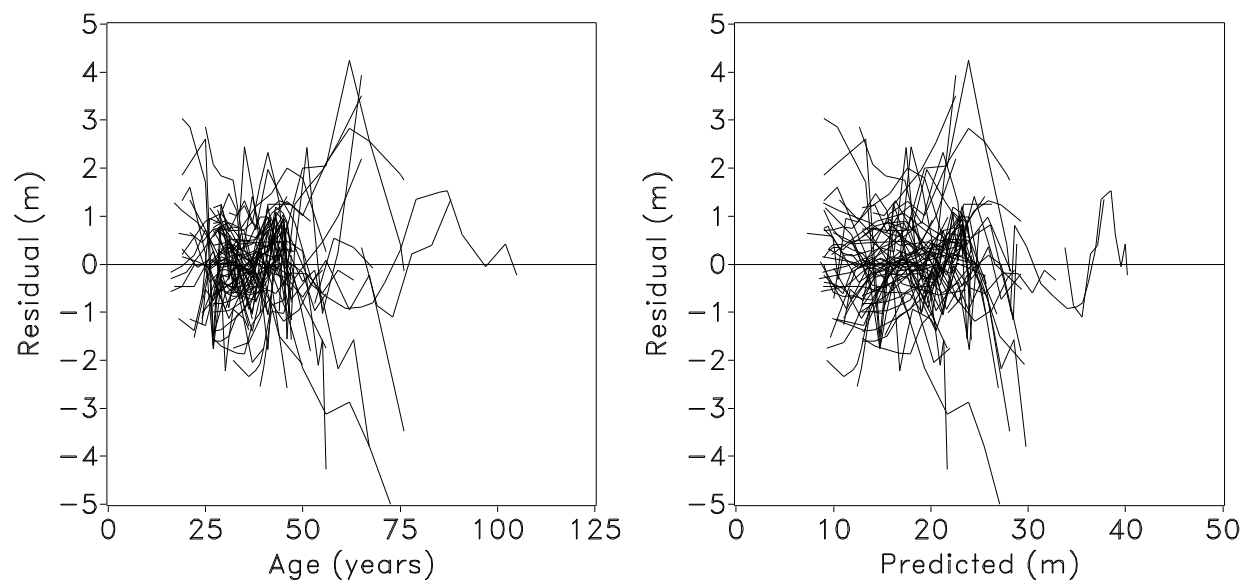


Figure 4.8: Residuals plotted against stand age and predicted dominant height for Sitka spruce.

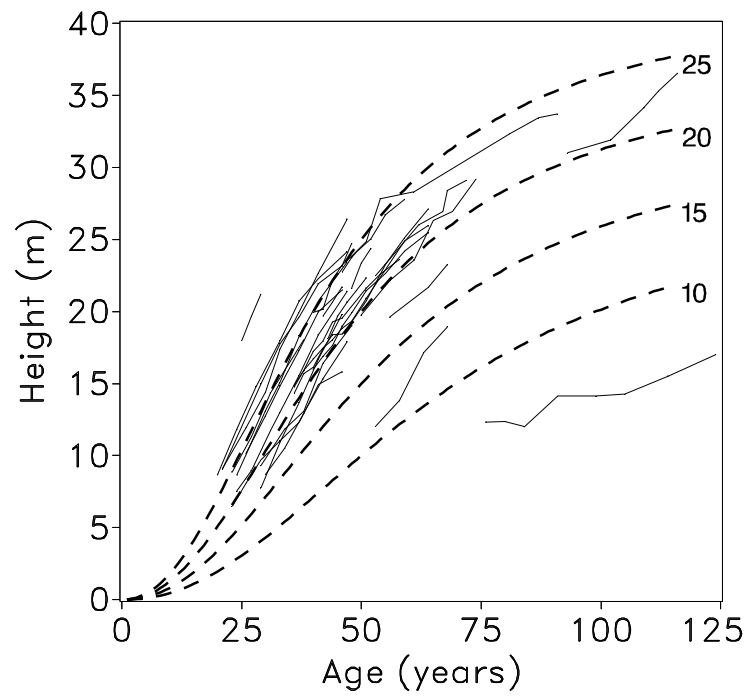


Figure 4.9: Site index curves for silver fir along with observed height-age patterns. Numbers refers to height at base age 50 years.

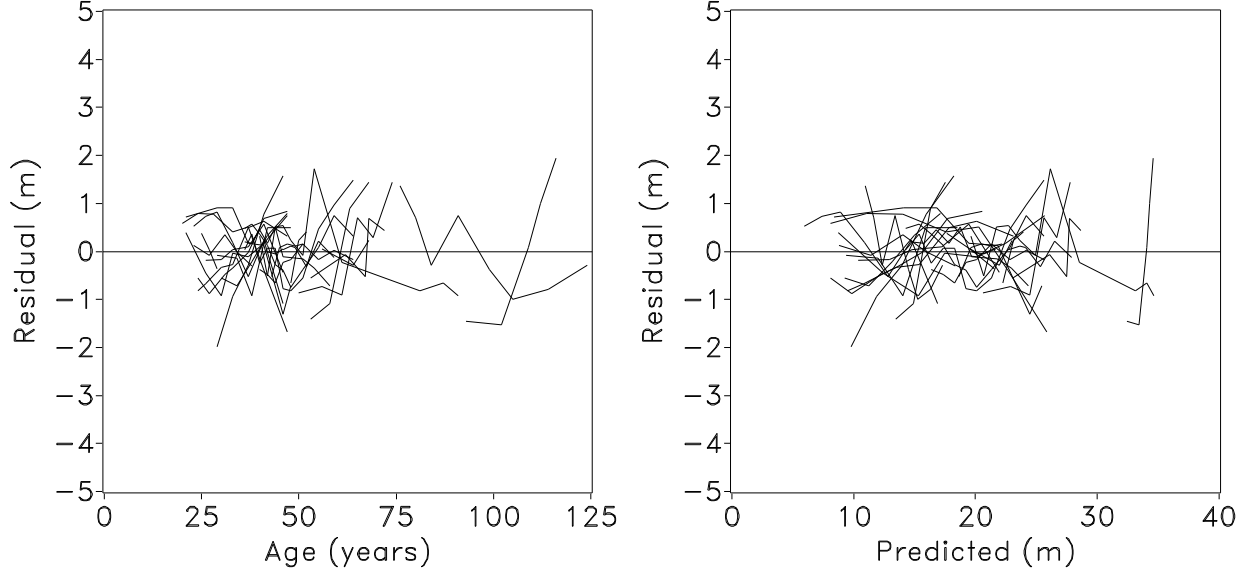


Figure 4.10: Residuals plotted against stand age and predicted dominant height for silver fir.

The site equations for silver fir explain more than 98% of the observed variation in dominant height and the residuals show no obvious trends across stand ages and predicted dominant heights (Figure 4.10).

4.2.6 Douglas fir

Douglas fir shows a very rapid height growth compared to the other species. Nevertheless, the chosen site equation seems to represent the observed age-height patterns satisfactorily (Figure 4.11). As For oak the parameter estimate of β_2 caused asymptotic height to decline for high site index values and the model collapsed. In this case β_2 was arbitrarily restricted to 1, resulting in an increase in RMSE of 0.3 %. Parameter estimates are provided in Table 4.6.

The site equations for Douglas fir explain close to 98% of the observed variation in dominant height and the residuals show no obvious trends across stand ages and predicted dominant heights (Figure 4.12).

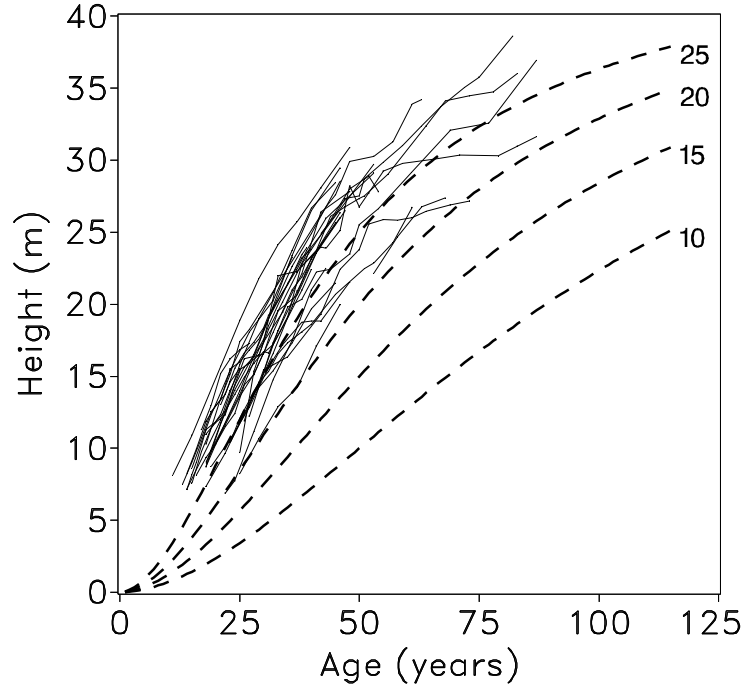


Figure 4.11: Site index curves for Douglas fir along with observed height-age patterns. Numbers refers to height at base age 50 years.

Table 4.6: Parameter estimates for the base-age invariant site equation for Douglas fir based on equation (4.1) (Cieszewski, 2001). The table includes fit statistics such as root mean square error (RMSE), R-square and average absolute bias (AAB).

Equation	Para.	Estimate	Std. Err.	t-stat	RMSE	R-square	AAB
(4.1)	β_1	1.8000	0.0766	34.65	0.8403	0.989	0.844
	β_2	1.0000	.	.			
	β_3	44.4553	2.2188	0.74			

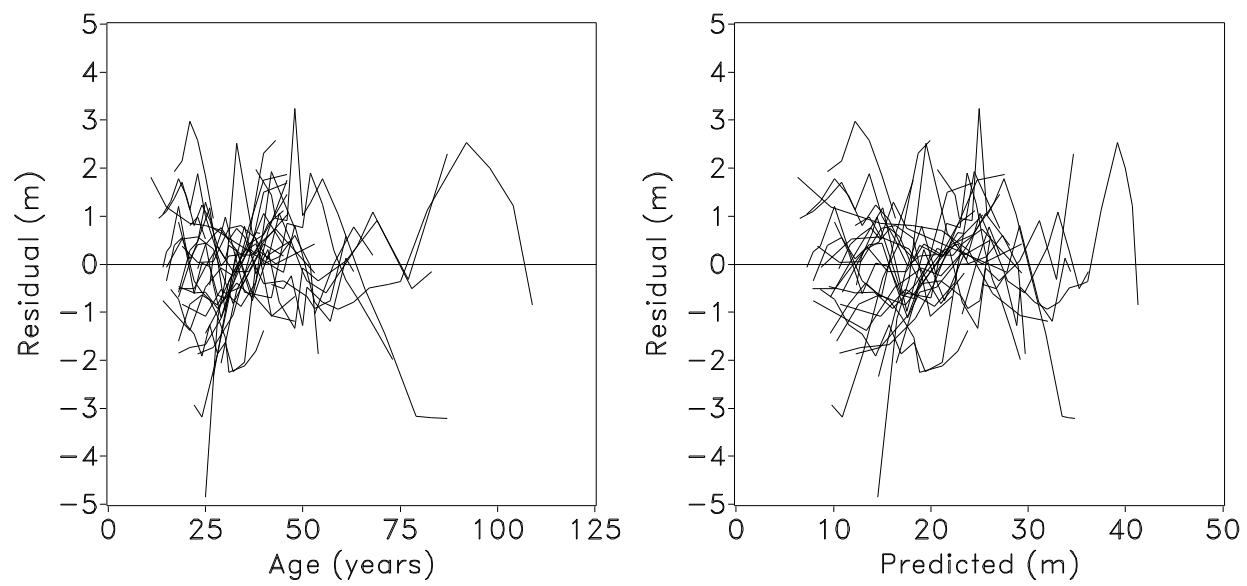


Figure 4.12: Residuals plotted against stand age and predicted dominant height for Douglas fir.

Chapter 5

Soil properties and growth

Site-specific parameters such as a_0 in equation (3.2.1)–(3.2.2) or site index are suited for situations where knowledge on growth can be obtained from the present or previous crops. However, in situations where the particular species has not previously been grown on the site, such measures cannot be obtained and therefore cannot be used in estimating growth. Other measures such as climate (precipitation, temperature), geography (soil type, elevation, parent material), or soil nutrient content have been suggested by a number of authors (Johannsen, 1999; Callesen, 2003).

Based on soil properties (fractions of clay, silt, fine sand, and coarse sand) we modeled the relationship between the site-specific parameter of the stand growth model (a_0 in equation (3.2.1)–(3.2.2)) and the fraction of coarse sand (in %) using a power function:

$$a_{0,i} = \alpha_1 \cdot (1 + CS_i)^{\alpha_2} + \varepsilon_i \quad (5.1)$$

where $a_{0,i}$ is the site-specific parameter of the i th site, CS is the coarse sand percentage, and ε is the random error.

5.1 Results

In general a negative relationship between coarse sand percentage and the site-specific parameter was observed (Figure 5.1) although the relation was generally weak (Table 5.1). Especially for beech and Sitka spruce the correlation between the two variables was so small that it makes little sense to estimate the site-specific parameter based on soil properties alone.

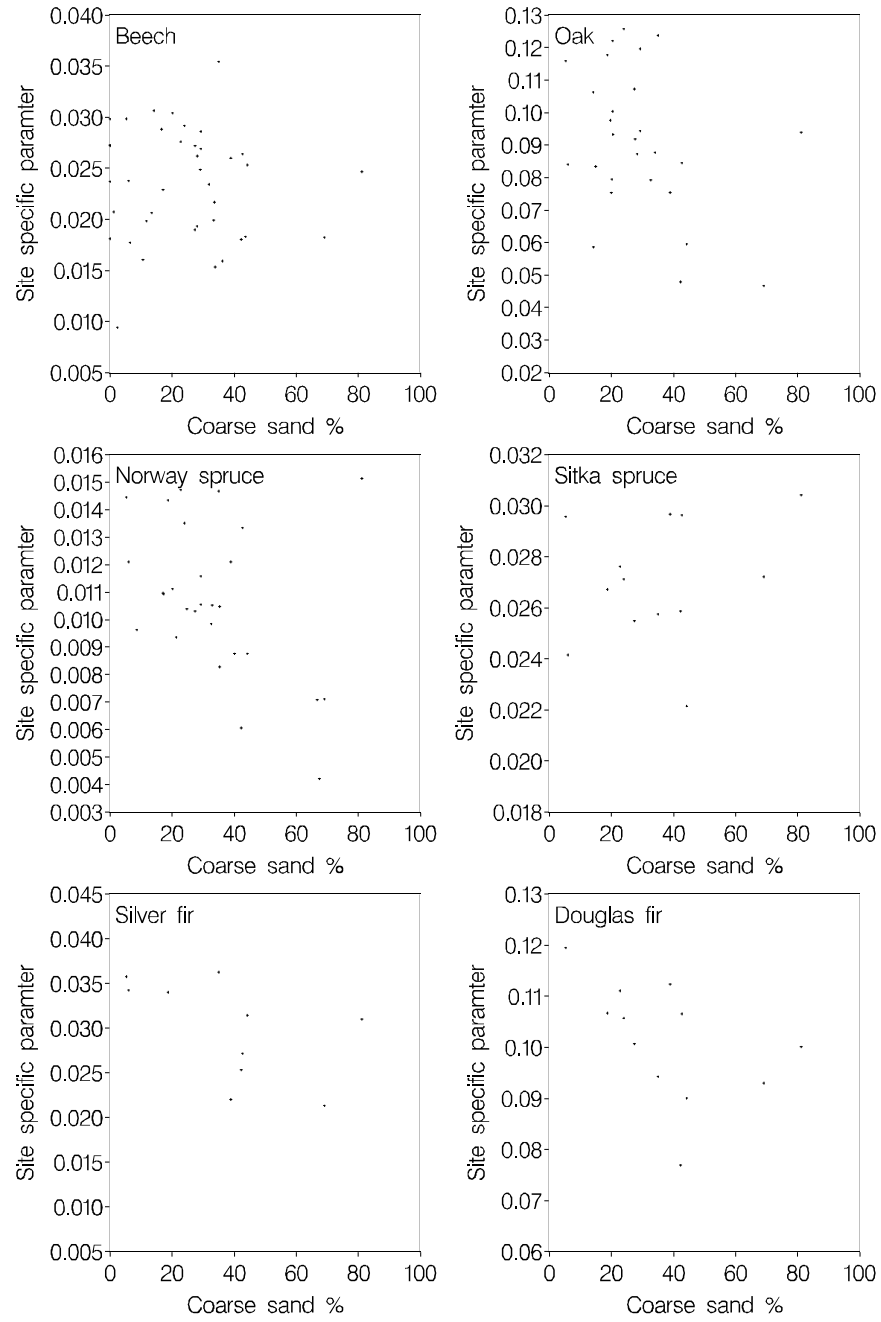


Figure 5.1: Relation between the site-specific parameter a_0 in equation (3.2.1)–(3.2.2) and the coarse sand percentage for beech, Norway spruce, oak, Sitka spruce, Douglas fir, and silver fir.

Table 5.1: Parameter estimates for the relationship between coarse sand percentage and the site-specific parameter a_0 in equation (3.2.1)–(3.2.2).

Species	Parameter		R-square
	α_1	α_2	
Beech	0.02348	-0.00043	0.000
Oak	0.13506	-0.12179	0.092
Norway spruce	0.01756	-0.14726	0.144
Sitka spruce	0.02564	0.01563	0.018
Silver fir	0.04474	-0.12012	0.359
Douglas fir	0.14411	-0.10136	0.378

Chapter 6

Relationship between height corresponding to quadratic mean diameter (H_g) and dominant height (H_{100})

Numerous variables may be used to characterize specific forest stands (basal area, stem numbers, average diameter, diameter distribution etc.), but few are so commonly used as stand height. Stand height is however not unequivocally defined as different definitions have proven adequate for different situations. A simple but rarely used definition of stand height would be a simple average of the tree heights. More commonly used for example in the common Danish yield tables is the height corresponding to quadratic mean diameter (H_g). This height is most commonly found by developing a diameter/height regression and subsequently calculating the height based on the quadratic mean diameter (D_g). The popularity of H_g is due to its relation to stand volume. A related measure of stand height is the basal area weighted mean height (Lorey's height, H_L).

Another commonly used measure of stand height is dominant height, often measured as the average height of the 100 thickest trees per hectare (H_{100}). The popularity of this measure of stand height is due to its properties as an indicator of site quality as it is less affected by thinning than for example H_g (except in the case of high thinning).

H_{100} has been used in most of the models underlying VIDAR (the dynamic growth models and site equations as mentioned above). However, as we wish to provide the user of VIDAR with the

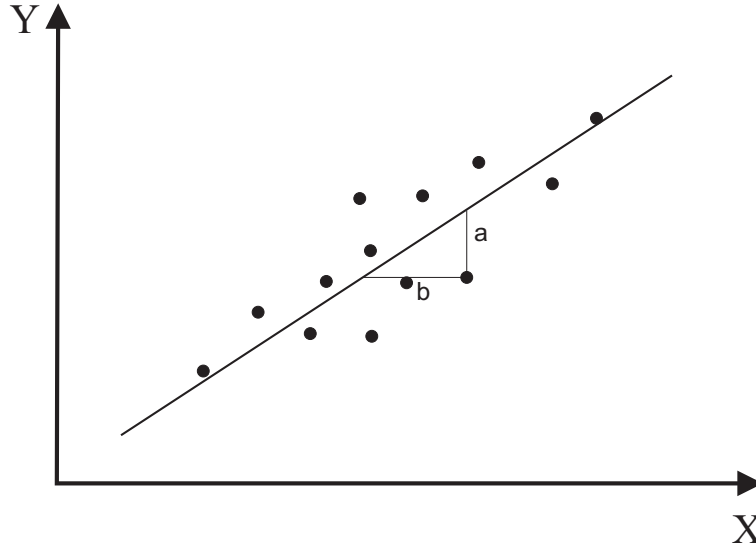


Figure 6.1: Illustration of linear regression.

highest possible degree of flexibility and to enable comparison with the existing yield tables we need to develop functions that transform H_g to H_{100} and vice versa. This chapter shows the development of such relationships.

6.1 Modeling the relationship between H_g and H_{100}

Development of the relation between H_g and H_{100} may seem as a relatively simple task as the relation between the two is both very strong and seemingly close to linear (figures 6.3, 6.5, 6.7, 6.9, 6.11), and 6.13.

Using linear regression, the regression line is found as the one that minimizes the squared vertical distances between the observed and predicted values (a in Figure 6.1). However, this is not the same as minimizing the squared horizontal distances between observed and predicted values (b in Figure 6.1). Hence, prediction of Y from an observation of X based on the linear regression illustrated in Figure 6.1 is unbiased, but rearranging the same regression to predict X given a value of Y is biased.

The problem of biased prediction when alternating between using H_g and H_{100} as the independent variable is highly relevant to the problem of developing a relationship between H_g and H_{100} , as we

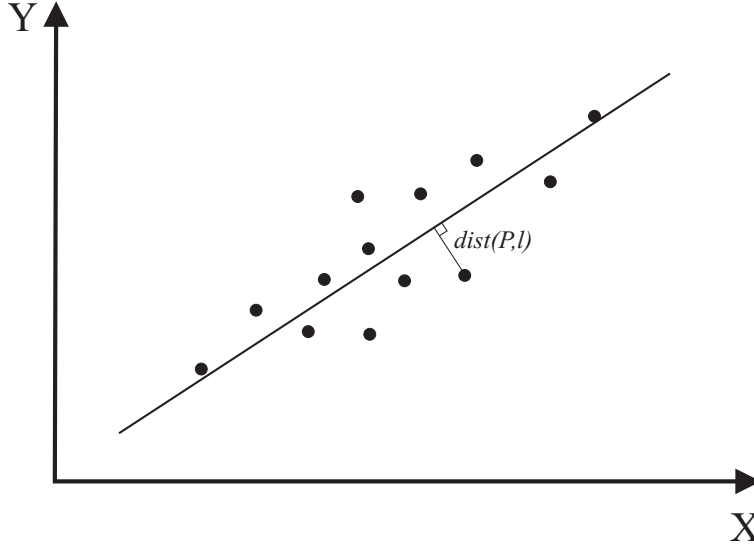


Figure 6.2: Illustration of linear regression.

are interested in predicting both H_g from H_{100} and vice versa. One way to deal with this problem is to develop separate regressions for predicting H_g from H_{100} and for predicting H_{100} from H_g . However, it is both unpractical as well as problematic because predicting H_g from H_{100} and vice versa will not yield the same concordant results. An alternative to this approach is to use orthogonal regression illustrated in Figure 6.2. Here the squared orthogonal distances between observations and the regression line are minimized. As this distance is the same whether the dependent variable is H_g or H_{100} , the relationship works equally well in both cases.

The orthogonal distance ($dist$) between a point $P(x_1, y_1)$ and the line l with the equation $y = ax + b$ is calculated by:

$$dist(P, l) = \frac{|ax_1 + b - y_1|}{\sqrt{a^2 + 1}} \quad (6.1)$$

Using the linear relationship between H_g and H_{100} :

$$H_{100,i} = a \cdot H_{g,i} + b + \varepsilon_i \quad (6.2)$$

we used the NLP procedure in SAS to minimize the squared orthogonal distance between (H_g, H_{100}) -observations and the regression line:

Table 6.1: Parameter estimates of the relation between H_g and H_{100} for beech, Norway spruce, oak, and Douglas fir.

Species	a	b
Beech	0.955129	1.746100
Oak	0.931169	1.898170
Norway spruce	0.980288	1.963037
Sitka spruce	0.968923	2.013658
Silver fir	0.944459	2.333214
Douglas fir	0.976051	1.592454

$$\min \sum_{i=1}^n \left(\frac{|ax_i + b - y_i|}{\sqrt{a^2 + 1}} \right)^2 \quad (6.3)$$

where x_i and y_i are the i th observation of H_g and H_{100} respectively.

6.2 Results

Parameter estimates of the regression $H_{100} = a \cdot H_g + b$ are provided in Table 6.1. Comments on the individual regressions are provided in the species-wise subsections below. A general observation is that the slopes are a little less than one, indicating that H_{100} is larger than H_g in young stands and that this difference is reduced through the life of the stand. The intercept is 1.5 – 2 m which is the difference between H_{100} and H_g for a hypothetical stand where $H_g = 0$.

6.2.1 Beech

The relation between H_g and H_{100} for beech was close to linear. The slope was, as expected close to 1 and the intercept was 1.7 m. These properties are in accordance with what can be observed from a graphical presentation of the relationship (Figure 6.3).

Residual plots of both H_g and H_{100} show that deviations of the predicted values from the observed are small (Figure 6.4). Note that the two plots are merely mirror images as predicted values are calculated by rearranging equation (6.2). RMSE for the prediction of H_{100} was 0.731 m and 0.765 m for the prediction of H_g .

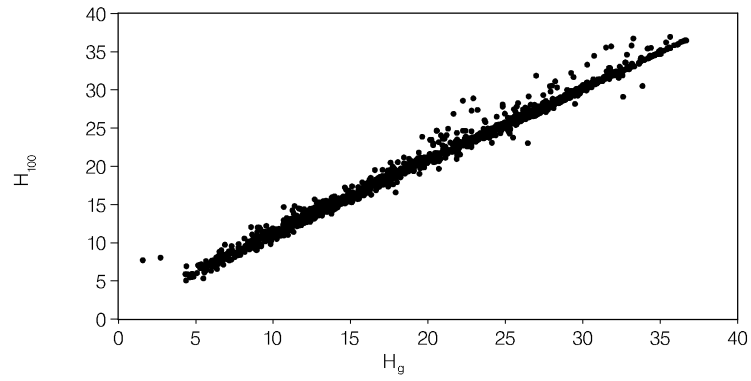


Figure 6.3: H_g vs. H_{100} for beech.

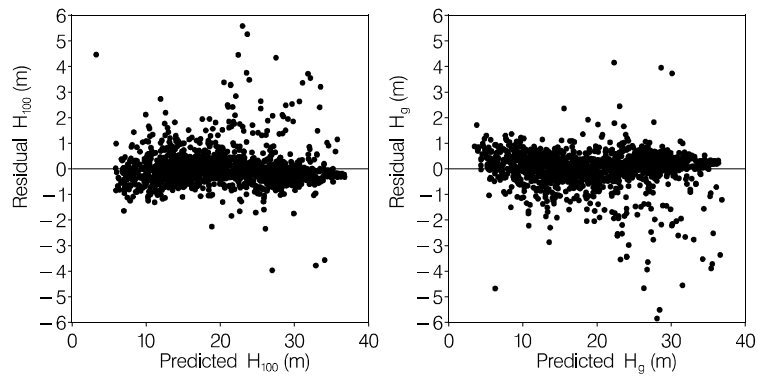


Figure 6.4: Residual plots of H_g and H_{100} for beech respectively.

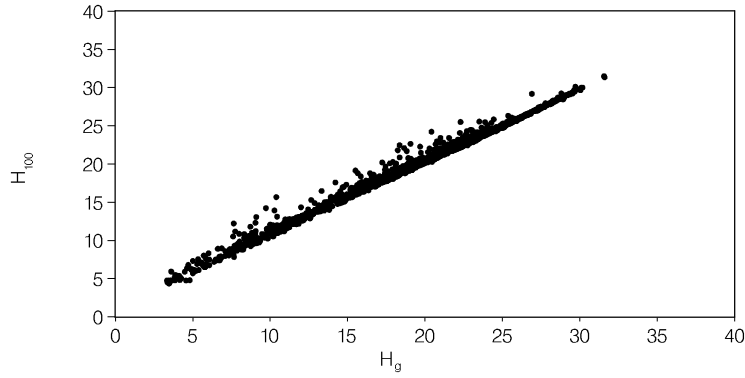


Figure 6.5: H_g vs. H_{100} for oak.

6.2.2 Oak

The relation between H_g and H_{100} for oak was close to linear. The slope was, as expected close to 1 and the intercept was 1.9 m. These properties are in accordance with what can be observed from a graphical presentation of the relationship (Figure 6.5).

Some systematic deviations from the model for oak was observed in old stands. As we minimized the squared deviations, the one-sided deviations affected the regression line which seems to have created a slight bias in the predictions (Figure 6.6). Despite the slight bias, RMSE for H_{100} (0.549 m) and H_g (0.590 m) were relatively small.

6.2.3 Norway spruce

The relation between H_g and H_{100} for Norway spruce was close to linear. The slope was, as expected close to 1 and the intercept was 2.0 m. These properties are in accordance with what can be observed from a graphical presentation of the relationship (Figure 6.7).

Residuals of both H_g and H_{100} are generally small (Figure 6.8). RMSE for the prediction of H_{100} was 0.743 m and 0.758 m for the prediction of H_g .

6.2.4 Sitka spruce

There was a very close to linear relationship between H_g and H_{100} for Sitka spruce (Figure 6.9). As with the other species the slope was, as expected, close to 1 and the intercept was 2.0 m.

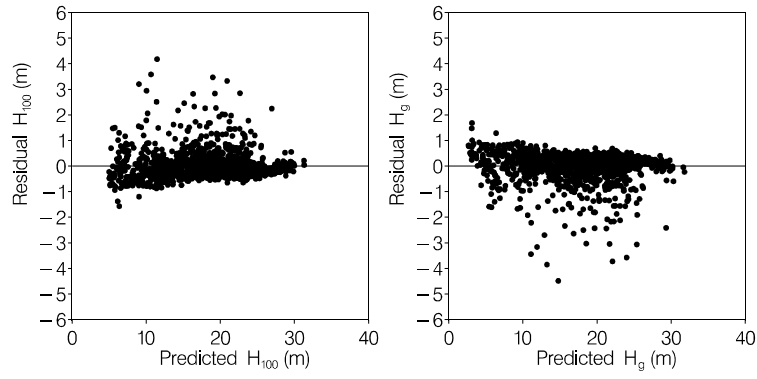


Figure 6.6: Relation between H_g and H_{100} for oak.

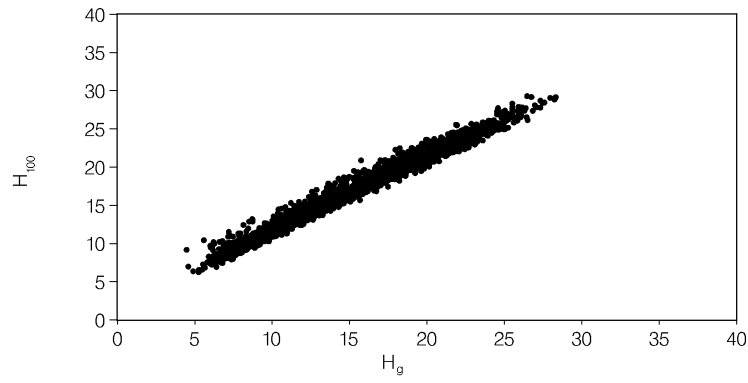


Figure 6.7: H_g vs. H_{100} for Norway spruce.

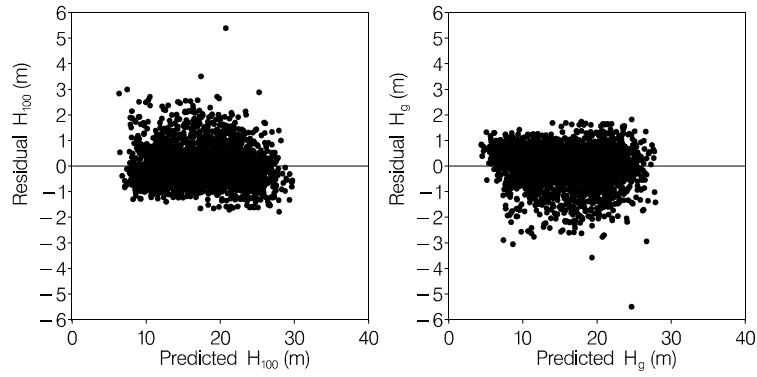


Figure 6.8: Relation between H_g and H_{100} for Norway spruce.

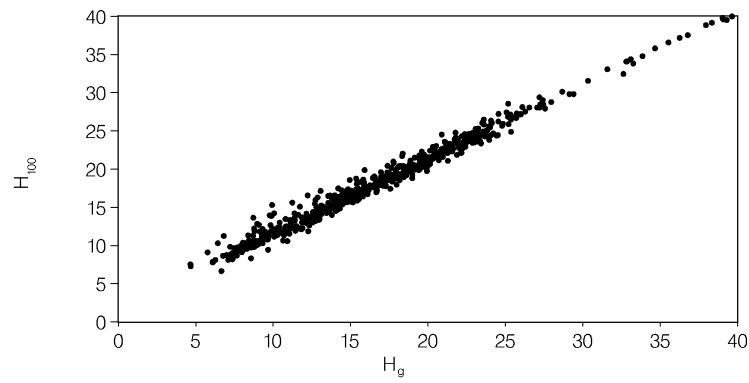


Figure 6.9: Relation between H_g and H_{100} for Sitka spruce.

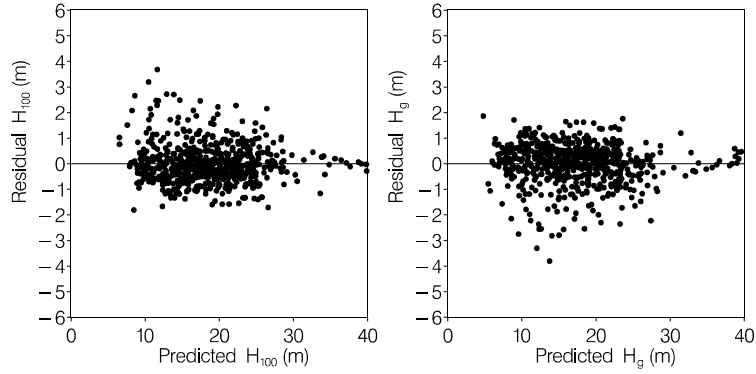


Figure 6.10: Relation between H_g and H_{100} for Sitka spruce.

Residuals of the relations are generally small with a RMSE for H_{100} and H_g of 0.768 and 0.793 m, respectively.

6.2.5 Silver fir

There was also for silver fir a very close to linear relationship between H_g and H_{100} (Figure 6.11). As with the other species the slope was, as expected close to 1 and the intercept was 2.3 m.

Residuals of the relations are generally small with a RMSE for H_{100} and H_g of 0.541 and 0.573 m, respectively (Figure 6.12).

6.2.6 Douglas fir

There was a very close to linear relationship between H_g and H_{100} for Douglas fir (Figure 6.13). As with the other species the slope was, as expected close to 1 and the intercept was 1.6 m.

Residuals of the relations are generally small with a RMSE for H_{100} and H_g of 0.562 and 0.576 m, respectively.

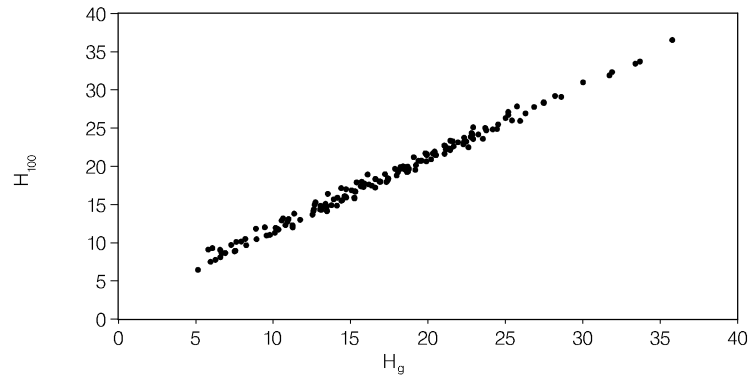


Figure 6.11: Relation between H_g and H_{100} for silver fir.

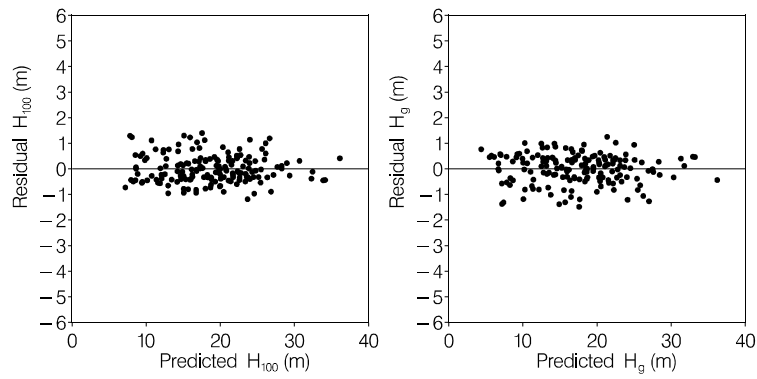


Figure 6.12: Relation between H_g and H_{100} for silver fir.

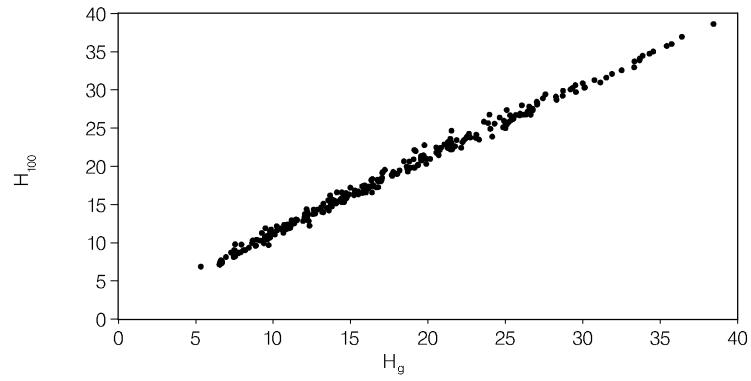


Figure 6.13: Relation between H_g and H_{100} for Douglas fir.

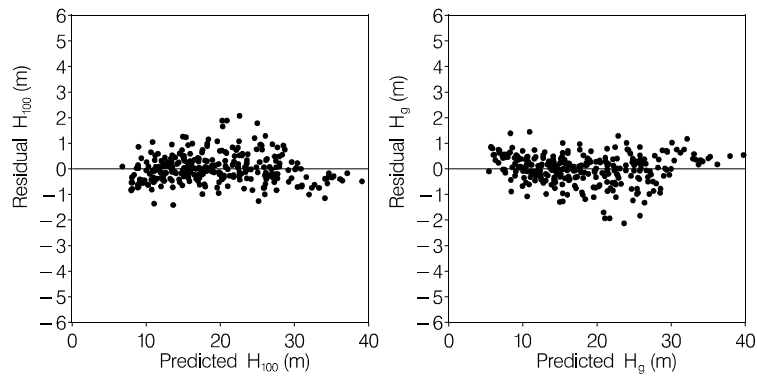


Figure 6.14: Relation between H_g and H_{100} for Douglas fir.

Chapter 7

Derivation of the relationship between harvest quotients for diameter and height

In most yield tables and other growth models it is assumed that stand height is unaffected by thinnings. This assumption is underlying the concept of site index and is generally believed to be valid in most situations. However, in VIDAR, the user is able to choose a wide range of thinning regimes. In this situation, the user may specify thinning regimes that certainly does affect stand height, such as high thinning, crown thinning, or heavy thinnings from below. Hence, the programme must be equipped with a module expressing the effect of thinning on stand height.

7.1 Modeling the relationship between Q_D and Q_H

We chose to model the effect of thinnings on stand height as the effect on the quotient of stand height before and after thinning:

$$Q_H = \frac{H_{g,3}}{H_{g,1}}, \quad (7.1)$$

where suffixes 1 and 3 refer to the stand height before and after thinning respectively. The effect of thinning on stand height was modeled as a function of the thinning quotient that expresses the relation between stand diameter after and before thinning. In VIDAR the thinning quotient is defined

as:

$$Q_D = \frac{D_{g,3}}{D_{g,1}}, \quad (7.2)$$

where suffix 1 and 3 refer to the diameter before and after thinning, respectively.

A graphical inspection of Q_H/Q_D -plots showed that the relation is increasing and has an inflexion point at (1,1) (Figures 7.1, 7.3, 7.2, and 7.6). Such relation may adequately be described by a third order polynomial, although this function does not have a well-defined inflexion point. We therefore restricted the polynomial to have inflexion point at (1,1):

$$Q_H = 1 - b - c + (c/3) + b \cdot Q_D + c \cdot Q_D^2 - (c/3) \cdot Q_D^3, \quad (7.3)$$

where:

$$Q_H = \frac{H_{g,3}}{H_{g,1}} \text{ and } Q_D = \frac{D_{g,3}}{D_{g,1}}$$

We estimated the regression coefficients using multiple regression analysis.

7.2 Results

The selected functional form generally fitted the data well for all species (Table 7.1). A general observation is that the observed range of Q_D is relatively narrow ($\sim 0.8 - 1.2$) and although the model may provide a good description inside this range, extrapolations using the model may be dubious.

Confidence intervals around the predicted regression line are relatively narrow for all species (Figures 7.1, 7.3, 7.2, 7.6, 7.4, and 7.5). It should however be noticed that residuals are far from homogeneous and thus that model inferences such as confidence intervals may not hold.

Table 7.1: Parameter estimates of the relationship between Q_H and Q_D for European beech, oak, Norway spruce, Sitka spruce, European silver fir, and Douglas fir. Regression coefficients are based on equation (7.3).

Species	Regression coefficients		Range	
	b	c	Q_D	Q_H
Beech	3.0975	-2.8248	0.28–1.25	0.45–1.12
Oak	0.2983	0.0182	0.94–1.49	0.97–1.16
Norway spruce	3.1025	-2.6356	0.59–1.33	0.81–1.19
Sitka spruce	5.9506	-5.5872	0.82–1.36	0.92–1.22
Silver fir	11.7545	-11.4020	0.97–1.28	0.99–1.11
Douglas fir	10.4952	-10.1946	0.93–1.20	0.97–1.09

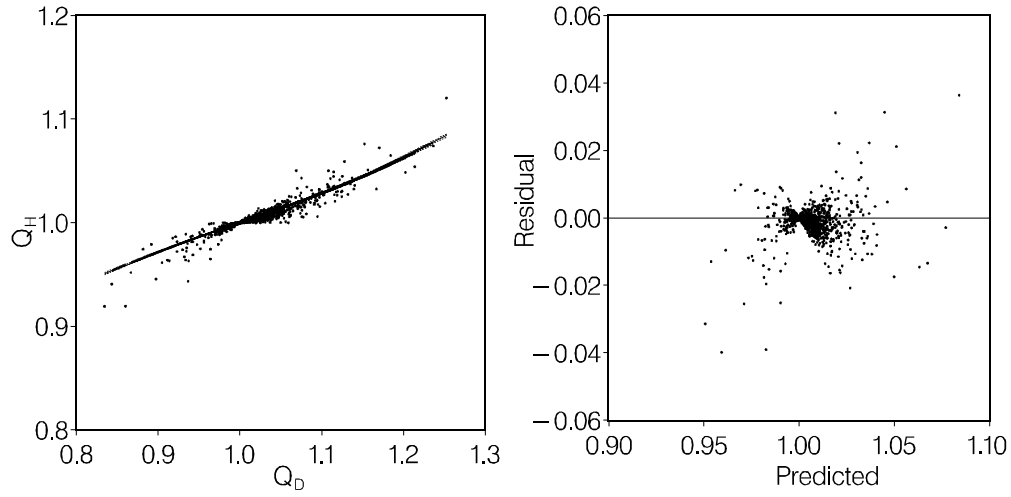


Figure 7.1: Relation between Q_D and Q_H for beech.

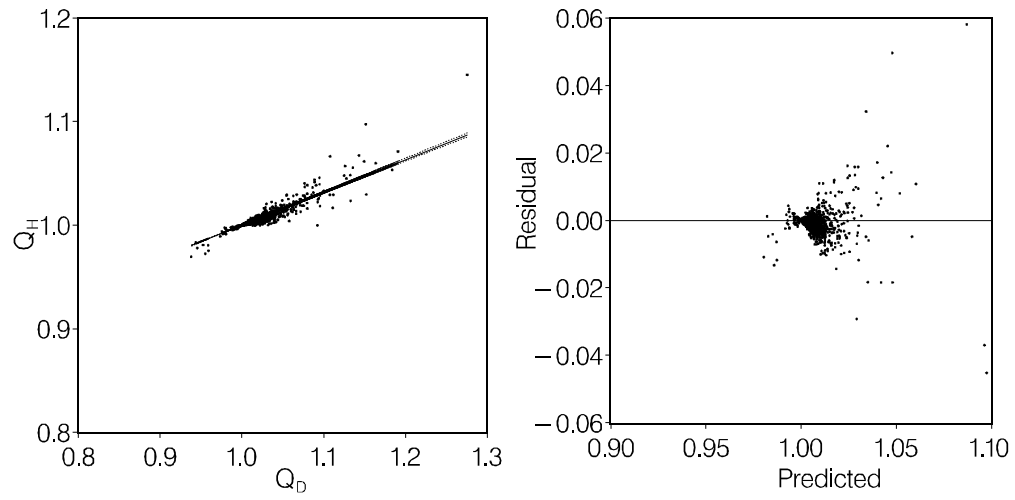


Figure 7.2: Relation between Q_D and Q_H for oak.

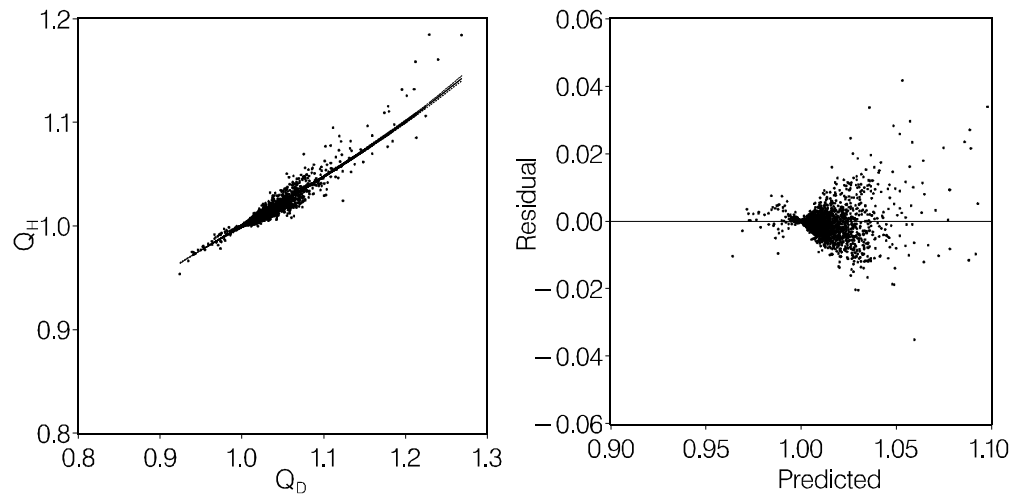


Figure 7.3: Relation between Q_D and Q_H for Norway spruce.

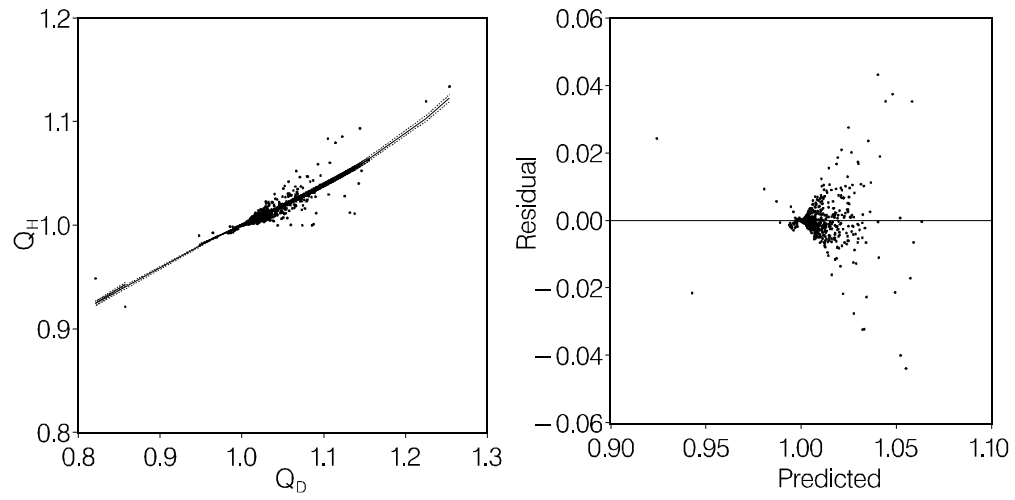


Figure 7.4: Relation between Q_D and Q_H for Sitka spruce.

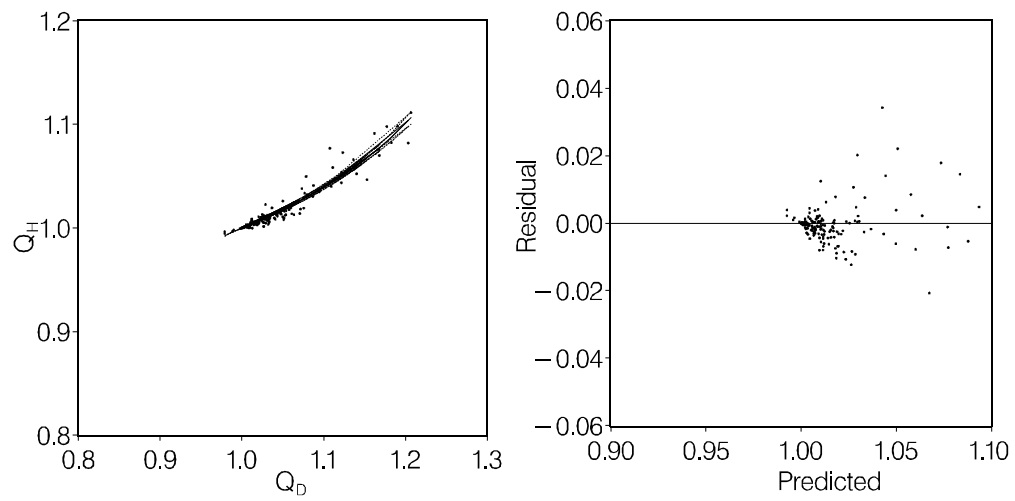


Figure 7.5: Relation between Q_D and Q_H for silver fir.

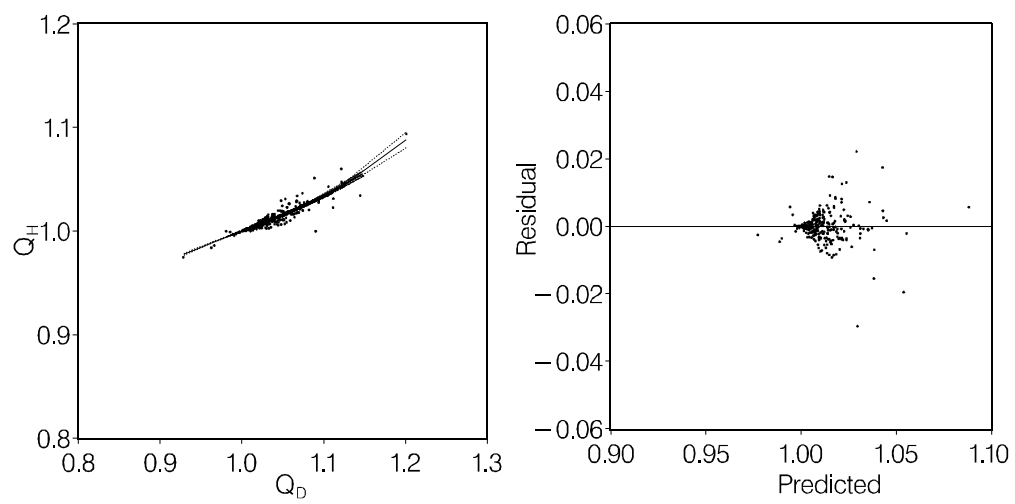


Figure 7.6: Relation between Q_D and Q_H for Douglas fir.

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